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Algorithm of multithreshold decoding for Gaussian channels [1]

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Abstract—Major principles of the method of the linear codes multithreshhold decoding as search for the global functional extremum for a great number of variables are considered. It was demonstrated that the multithreshold decoding efficiency is close to the results ensured by the optimum exhaustive search methods. Decoding complexity under software and hardware realizations is evaluated.

1. INTRODUCTION

As is known, the use of error-correcting coding enables to solve a variety of tasks in digital networks, which are in principle inaccessible under analog signal processing. The main advantage of communications systems using coding amounts to the fact that the efficiency of the channels' use, i.e. their efficiency coefficient, appear to be many times as high as in cases when codes are not used. The coding gain is usually chosen a measure of efficiency. It shows to what degree it is possible to decrease the specific energy of the channel, i.e. the ratio of data bit transmission average power to the noise power spectral density E_b/N_0 with usage of certain coding and decoding methods as compared with the case without their usage to ensure maintenance of the high degree of transmission validity, which is essential in this system. For example, the required bit error rate (BER) shall amount to 10^{-6}. It is by this potential that the necessity of coding use is predetermined for systems of space, satellite and mobile communications.

The ability of codes to ensure highly reliable data transmission with a low signal level enables to minimize the hardware dimensions, increase speed of transmission via most expensive digital communications channels, cut on the antennae dimensions substantially and increase the life time of self-contained power supply manifold. At that coding gain can reach 10 dB and more in many cases. Apart from that, it is important to realize that the necessary reliability of the data transmission in channels with noise and digital data storage systems is only to increase with time, which, other things equal, will result in toughening of requirements to the coding systems. This will in turn contribute to the growth of the provided coding gain, which should be realized in the least expensive way, i.e. on the basis of efficient codes and indispensably very simple and fast decoders.

Both decoders realizing the well known to specialists Viterbi algorithm (AV) [1], and also much more complex code structures, such as turbo codes [2] and low-density parity-check codes [3], are currently being used in digital communications. Nevertheless, the currently used coding systems, especially those for high-speed channels, are still very complex and inefficient. Under consideration bellow are the theoretical basis and concrete parameters of a highly efficient, high performance and very simple iterative algorithm of error correction in high noise level channels, which is the result of the development of the concept of linear convolutional codes majority decoding [4].

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2. ITERATIVE DECODING PRINCIPLE

In the past decade the theory and technology of error-correcting coding has progressed greatly. The introduced by many authors in the seventies of the twentieth century methods of iterative decoding of accepted messages turned out to be inefficient due to powerful error grouping at the decoder output. An example of such a scheme with a threshold decoder (TD) [4] for a convolutional code is given in Fig. 1.

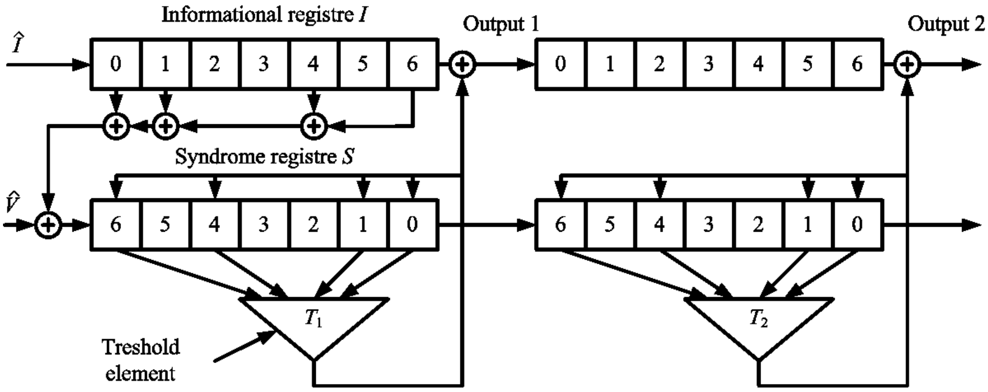

Figure 1. An example of a scheme of iterative decoding on the basis of the convolutional code threshold decoder

Low efficiency of the aforementioned decoding scheme stemmed from intense grouping, i.e. propagation of errors at the threshold decoder. In fact, if under a certain noise level in a binary symmetrical channel with independent errors the TD has at a certain moment taken a wrong decision on the next information symbol, a very dense error packet usually emerges at the output of that TD. For example, let us assume that a sequence, improved to a certain extent after the first decoding attempt, has come from the first TD output in Fig. 1 to the input of the second one. Then, if there are no errors in a certain part of the information sequence after the first TD, there is no need for a second decoder. But when an error emerges at the output of the first TD, which is usually a starting point for the typical error packet of this TD, it appears that the second decoder, copying exactly the scheme of the first one and tuned only to random errors correction, will most probably fail to correct the packet. Hence there is no need to use it in this case.

It should be noted that codes with low level of error propagation in TD were completely unheard of in those days. Nevertheless, the problem was solved in full later on with the help of methods described in [5]–[15]. In this connection of great importance appears to be the considered further on new approach to the realization of simple efficient error correction procedures, which is under development since 1972 and is called multithreshold decoding (MTD) [5].

3. GLOBAL FUNCTIONAL OPTIMIZATION PRINCIPLE

Surprisingly the development of methods of error-correcting codes decoding has not for a long period of time been in any way associated with methods of solving tasks of functional optimization of many discrete variables. Nevertheless, it should have been quite natural to consider the decoding, i.e. search of a single code word out of an exponentially great number of possible messages, proceeding from this concept. However, the majority of the previously developed decoding algorithms never used for the search of the best decoder decisions the well known and manifold powerful optimization procedures, which could be reasonably applied to the search of code words located at a minimum possible distance from the accepted message. It should be noted, that the widely applied in communications engineering Viterbi algorithm, used for decoding by maximum likelihood of short convolutional codes, does not refer to the optimization procedures class either, as it is directly searching for an optimum solution on the basis of a very convenient for realization full enumeration method.

Alongside with that, certain decoding algorithms, specifically, threshold decoders [4], being the simplest error correction methods, are almost possessing the properties needed for the realization of full-fledged, efficient and at the same time exceptionally simple iterative decoding procedures, which would really have been the methods of the search for the functional global extremum of a very great amount of variables.

To substantiate this let us consider an example of the simplest system of coding/threshold decoding with the code rate $R = 1/2$ and the minimum code distance $d = 3$, as shown in Fig. 2.

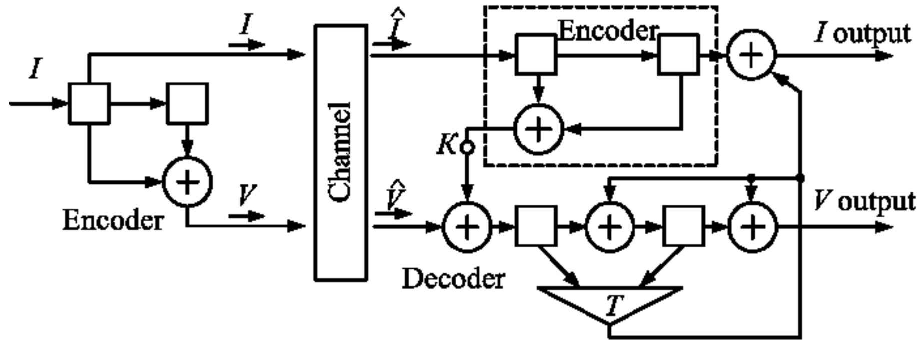

Figure 2. Special projection of the coding system clarifying the new interpretation of the syndrome vector

As follows from the coder type and the simplest majority decoder correcting in this simple example a single error, the decoder comprises an exact copy of the coder, which is forming its estimates of the code-checking symbol on the basis of the received via the channel code information symbols (possibly containing errors). These symbols appear at the decoder K point and after the adding on the half-adder with the received from the channel checking symbols $\widehat{V}$ form symbols of the S syndrome vector, which is dependant only on the channel errors vector. Later on these symbols come to the decoder threshold element $'$ from the syndrome register, as is shown in the Fig. 2.

The very form of the TD given in the scheme of coding/decoding enables to single out an easy way of the organization of the proper optimization procedure, i.e. search for the best possible decision during decoding. To that purpose let us stress the fact that has never been mentioned before: in the decoder syndrome register there is a difference by check symbols between the received with distortions from the channel $Q = [\widehat{I}, \widehat{V}]$ vector and such a code word A_r, the information symbols of which coincide with the received from the channel information part of vector Q.

It means that the full difference between the code word, the current hypothesis-decision of decoder A_i on the transmitted code word, and the received noisy vector Q will be in such a modified decoder of a majority type; only one vector will be added to the TD, and the vector shall always comply with the difference between the received vector Q and A_i, the current hypotheses of the decoder on the information symbols. It is this decoder that is going to have the full value of the difference, and, consequently, the full distance between the decision of the decoder and the received vector. One should strive at decreasing this distance to the minimum possible value, which is to correspond to the decision of the optimum decoder (OD).

4. THE MTD OPERATION PRINCIPLE

It is this very approach to the problem of high efficiency decoding that is the basis of the developed since 1972 special iterative multithreshold decoders [5]–[15], almost coinciding with the classic TD and as simple in realization as their prototype.

The changes that should be introduced into ordinary TD to transform it into MTD (as follows from the global optimization principle discussed in the previous section), amount only to the fact that the decisions of all the threshold elements on the decoded symbols changes are first committed to memory in the additional differential register D, primarily, naturally, a zero one. These decisions are later on used by the following threshold elements of the decoder as an additional checking procedure in the course of further specification of the decoded symbols' meanings. Such a decoder is already measuring full distances between newer and more likely potential solutions and the received vector Q. It changes the decoded symbols in such a way that every new decision of such MTD is always closer to the vector received from the channel. In many instances it enables to practically completely realize the corrective potential of the used codes. Examples of concrete MTD schemes are given in [9, 10].

After such a rather unsophisticated improvement the decoder obtains new very useful properties. The MTD decisions at every change of the information symbols coded by it are strictly approaching the optimum decoder's decision ensuring in many cases the realization of this process even after several dozens of attempts of the correction of code block or the symbol flow of the convolutional code. Certainly, to ensure high efficiency of MTD under heavy noise in the channel it is essential to choose only special codes with low levels of error propagation. This important issue was considered in [6, 8, 9].

Let us further proceed with a more formal consideration of the MTD potential.

5. THE MTD MAIN THEOREM

Let a binary linear systematic block or convolutional self-orthogonal code [16] be set with the code rate $R = k/n$, where k is a number of information symbols, n is the length of the code combination.

During the transmission via memoriless BSC the optimum decoder minimizing the mean error probability, chooses from a set 2^k of equiprobable code words $\{A\}$ a vector A_0, for which the Hamming distance $r = |Q \oplus A|$, where Q is the received message, $\oplus$ − addition mod 2, would be minimum for the whole set $\{A\}$.

Let us represent any binary code X of the n length by a pair of vectors X_I and X_V of k length and $(n-k)$ referring to the information and check parts of the vector respectively:

$$X = [X_I, X_V].$$

Then, with the assumption that the parity-check matrix is represented systematically ($H = [C, I]$), we have the following

Lemma. *For each code vector A and received message Q the ratio is true*

$$A \oplus Q = [D, H[Q_I \oplus D, Q_V]], \tag{1}$$

where vector D of the length k is defined by the ratio

$$A_I = Q_I \oplus D. \tag{2}$$

Proof. Due to the linearity of the code

$$S = H[Q_I \oplus D, Q_V] = H[A_I, A_V \oplus A_V \oplus Q_V] = HA \oplus H[0_I, A_V \oplus Q_V],$$

where 0_I – is a zero information word.

As $HA = 0$, with A being the code word, and $H[0_I, A_V \oplus Q_V] = A_V \oplus Q_V$, as $A_V \oplus Q_V$ is multiplied only by identity submatrix I of the Hmatrix, we get that vector S is equal

$$S = A_V \oplus Q_V. \tag{3}$$

After substitutions in the right-hand side (1) with view of (2) and (3), we find that

$$[D, S] = [D, A_V \oplus Q_V] = [D \oplus Q_I \oplus Q_I, A_V \oplus Q_V] = A \oplus Q.$$

Thus the syndrome S vector is actually (as it was given in Fig. 2) a difference by checking symbols between the coming from the channel partially distorted message and the code word defined above. **The lemma is proved.**

The essence amounts to the fact that the difference $B = Q \oplus A$ for any received vector Q and code word A is defined by a pair of vectors $[D, S]$. Using exhaustive search of all the A vectors one is able to find vector A_0, minimizing $|B|$ and being the optimum decoder solution. By definition with $D = 0$ vector S is a usual syndrome of the received message Q: $S = HQ$. For simplicity in what follows we shall hereinafter and with $D \neq 0$ call S a syndrome as the generalization seems natural and is not resulting in any contradictions later on. It should be also noted that there is no need to calculate anew all the syndrome components with each change of A. It appears quite sufficient to invert, at each change increment, only the components S with odd amounts of errors in the changed information symbols. Nevertheless, exhaustive algorithms are most complex.

For this reason let us consider a decoding algorithm, which is very close to the threshold one, and is, therefore, easily realizable.

1. Let the decoder at the first preparatory stage perform calculation and memorizing of vector S. After that the realization of the decoding procedure per se begins.
2. A certain information symbol ij is chosen and for that symbol the usual sum of the syndrome s_{j_k} components is calculated, having as addends error e_j in the decoded symbol i_j (i.e. sum of checks $s_{j_k} \in \{S_j\}$, where $\{S_j\}$ is a set of checks as regard to component e_j, corresponding to symbol i_j) and symbol d_j, of the component of vector D, also referring to the decoded symbol i_j:

$$L_j = \sum_{s_{j_k} \in \{S_j\}} s_{j_k} + d_j. \tag{4}$$

   Let us at that assume that originally $D = 0$, because prior to the decoding operation commencement there is only one received vector Q in the memory of the decoder and the decoder possesses no other more preferable hypotheses of the received message.

   Let us assume the T threshold as equal to a half of all the addends in (4). For self-orthogonal codes the value amounts to $T = d/2 = (J + 1)/2$.
3. Let finally all the $J = d - 1$ of checks, i_j and d_j be inverted with $L_j > T$ and stay unchanged with $L_j \leq T$.
4. Provided no decision on the cancellation of the decoding procedure is taken the decoder reverts to step 2.

For the first decoding attempt the proposed procedure (as long as $d_j = 0$, is similar to the usual algorithm for a TD. Let us hereinafter refer to the decoder realizing the proposed algorithm as a multithreshold decoder (MTD).

Theorem (The main theorem of multithreshold decoding). *If at a random j-step MTD changes the currently being decoded information symbol i_j, then:*

a) at that MTD finds a new code word A_2, closer to the received message Q, then code word A_1, to which the i_j meaning corresponded prior to the j-th step of decoding

$$|B_1| = |A_1 \oplus Q] > |A_2 \oplus Q| = |B_2|;$$

b) after the completion of the j-th step decoding of any subsequent symbol i_k, $k \neq j$, is possible, so that its change will result in further approximation to the received message.

Proof. Prior to the decoding of symbol i_j it is true pursuant to Lemma

$$[D_1, S_1] = [A_{1I} \oplus Q_I, H[Q_I \oplus D_1, Q_V]] = A_1 \oplus Q,$$

where

$$A_1 = [A_{1I}, A_{1V}], \quad A_{1I} = Q_I \oplus D_1.$$

The weight of vector B_1 before this step, amounting to $|B_1| = |D_1| + |S_1|$, can be defined as an ordinary sum of weights $W_1 = L_{1j} + X$, where L_{1j} is defined by (4) and is equal to the sum of checks and symbol d_j at the threshold element; X is the weight of the other components S_1 and D_1, not included into L_{1j}.

Let us consider code vector A_2, differing from A_1 only in one information symbol i_j, and the respective difference $B_2 = A_2 \oplus Q$. As B_1 and B_2 differ only in the components coming to the threshold element $|B_2| = L_{2j} + X$, where $L_{1j} + L_{2j} = J + 1$,as due to the code's linearity, each check and the d_j are surely equal to 1 in only one of the two vectors B_i.

As MTD is changing i_j, if $L_{1j} > T$, it is essential for that to have $L_2 < L_1$ and, consequently, $|B_1| > |B_2|$, which proves item a) of the theorem.

It is further evident that if symbol i_j was not changed it is possible to decode any other symbol i_k, $k \neq j$, as at that the conditions of Lemma are held. In case of a change i_j in accordance with the rules of the MTD functioning after decoding i_j equations $A_{2I} = Q_I \oplus D_2$ and $S_2 = H[Q_I \oplus D_2, Q_V]$ hold, where D_2 differs from D_1 in symbol d_j, as in the course of changes via feedback from the threshold element of checks referring to i_j, the very components of S_1 are inverted, in which S_2 differs from S_1. Hence, we get that, after changing i_j for the previously defined vectors D_2, A_2 and S_2 the equation holds

$$[D_2, S_2] = A_2 \oplus Q,$$

similar to the one, occurring prior to the change of i_j. Thereby with subsequent decoding steps and changes of symbols i_k, $k \neq j$, there will be further approximation to the received form the channel message Q. **The main MTD theorem is proved.**

It follows from the theorem that with each change of the decoded symbols MTD is getting closer and closer to the received vector Q, thus finding closer and closer to the optimum decision and more likelihood vectors A_i. The MTD keeps viewing and matching not an exponentially great amount of code words but only pairs differing only in a single information symbol with one of the matched words being in the decoder. In case the second code word turns out to be closer to vector Q, than the one in the MTD, the decoder will switch over to that word to perform further matching with the new intermediate vector A_i. It is clear that in principle it is possible to carry out a large number of decoding attempts of all the code symbols. In that way convergence to the optimum decoder decision vector A_0. will be realized. It is crucial that for all that the MTD complexity remains the same as for the customary TD: a linear one.

Let us further assume that the MTD has reached the optimum decoder's decision, i.e. there are symbols of vector A_0. in the MTD information register. Then it is true that:

Corollary. *The MTD is not going to change the decision of the optimum decoder.*

Proof. If the MTD had changed during a certain step even a single information symbol in vector A_0, that would mean that another code vector A^* was found, which was closer to Q, than A_0, which is impossible, because by definition the closest to Q word is vector A_0. **The corollary is proved.**

Thus, the stability of the MTD decision on the optimum decision: having reached that, the MTD is going to stay there. It is very important as the algorithm implies an opportunity of multiple changes of the decoded symbols.

It might also be noted that during the proving of the main MTD theorem the uniqueness of the decoded symbol i_j was not used in any meaningful way. It follows that the aforesaid decoding procedure can be applied to any group of information symbols [6, 8, 9].

To apply the MTD algorithm while decoding in a channel with additive white Gaussian noise (AWGN) with quantization of the received binary stream into M levels, $M > 2$, , it is convenient to present the likelihood function L_j as

$$L_j = \sum_{s_{j_k} \in \{S_j\}} w_{i_k}(2s_{j_k} - 1) + w_{d_j}(2d_j - 1). \qquad (5)$$

For an ordinary BSC this expression with check weights w_{j_k}, amounting to 1, is evidently equivalent (4). With transfer to a Gaussian channel, i.e. in case of $M > 2$ signal quantization levels, weight coefficients for the calculation of L_j may be chosen as relatively small real or integer numbers. Thus, the symbols decoded in the MTD for a Gaussian channel should be changed with $L_j > 0$. At that, if $M \gg 1$, then, as is known, the corrective potential of the used codes and good algorithms of their decoding, MTD included, are usually improved by about 2 dB by the signal/noise ratio at the decoder input.

6. DECODING ERROR PROPAGATION IN MAJORITY DECODERS

It follows from the results we have proved above that the increase of the number of attempts to correct the symbols decoded before with the help of the MTD might be actually of use, as with every change of the information bits there is a transfer to decisions with higher likelihood. Nevertheless, it does not mean that the MTD is sure to arrive at the optimum decision. For many codes there exists a rather numerous amount of channel errors combinations, which are corrected in the course of optimum decoding but not corrected with the help of MTD. To a considerable extent it occurs due to the fact that threshold decoders are to a great extent subject to the influence of the error propagation effect. The second and the rest subsequently connected improved TDs, which comprise, for example, a convolutional MTD, usually have to operate mostly with streams of error packets from prior iterations of the decoders' decoding.

In [6, 8, 9] a method of estimate of error propagation for self-orthogonal codes is given [16], it amounts to the concept that probability estimates for the emergence of single errors and packets at the TD output are calculated using multidimensional probability generating functions. This method is helpful both for the selection of code in the least degree subject to error propagation influence, and for the choice of optimum weights and thresholds in the MTD ensuring the least probability values for its decoding errors.

A basis for a new approach to error propagation estimates is a rather convenient way of estimating the probabilities for the emergence of two errors within the constraint length or within each code block. It enables to generalize this method to comprise decoding error packets of any weight. To ensure high efficiency of the MTD it usually suffices to consider packet weights not exceeding 3. At that one has to calculate within the parameter space with the number of dimensions 2^{3d}, where d is the minimum code distance of the code. But for codes with $d \sim 7$ and more this task is too complex for computations.

Nevertheless, in the course of further investigation, methods of considerable simplification of estimates of packets emergence probability were found, which later on enabled to formulate a complex of criteria for future creation of codes with very low probabilities of the emergence of error packets during majority decoding. The respective algorithms for the development of such codes with the length n require realization of the order n^4 operations, which enables a search for efficient codes with lengths of the order of 500000 bits. Recently these algorithms were improved further.

While decoding close to the channel capacity it is essential to use only very long codes. That is why the completed development of constructive methods of the creation of codes with the required quality solved in full the problem of choosing codes with modest error propagation level for high efficiency decoders of the MTD class.

In the following sections we shall consider parameters of different decoders of the multithreshold type.

7. BASIC MTD PERFORMANCE

The dependence of BER on the signal/noise ratio E_b/N_0 in a channel with AWGN with the use of a soft-decision modem for a multithreshold decoder of self-orthogonal codes with the code rate $R = 1/2$ and different code distances d is given in Fig. 3. The dot line shows a rather exact estimate of the decoding BER for the same codes using an optimum decoder. The given graphs show that MTD actually ensures close to optimum decoding of the correctly chosen codes with a rather high noise level in the communications channel.

8. THE MTD ALGORITHM REALIZATION COMPLEXITY

The main MTD's advantage is the extremely low complexity of decoding. As in case with a traditional TD, at each iteration weighted checks are summed in the MTD; these are further on compared with the threshold and changed together with the decoded symbol provided the threshold is exceeded. The number of iterations of decoding I in this case is no more than 50, and the general MTD decoding complexity is evidently estimated for $d < 25$ as

$$N_1 \sim (d + 2)(I + 4).$$

But it is possible to decrease considerably the amount of iterative sum calculations at the threshold as the symbols at each of the threshold elements change in the course of the decoding process very seldom. If with the same conditions at I and d the decrease of the MTD performance is possible only by 0,1 dB in energy, which is usually quite acceptable, the amount of calculations will be decreased even more:

$$N_2 \sim c_1 d + c_2 I,$$

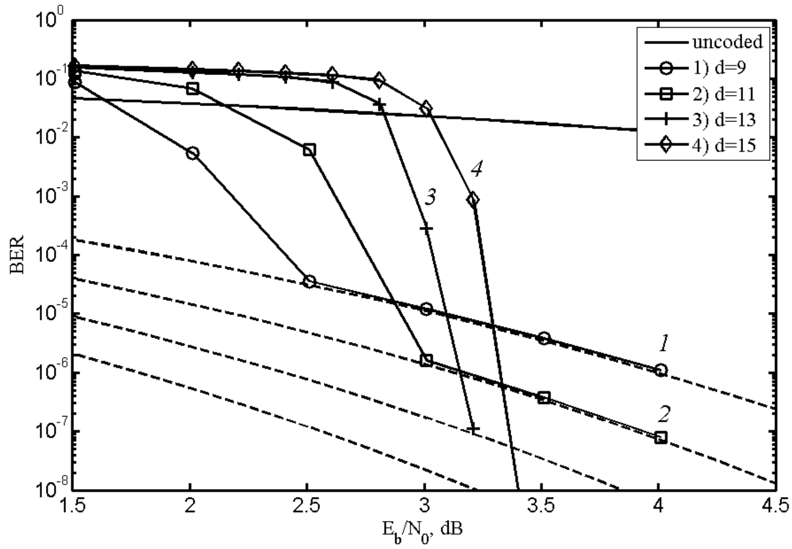

Figure 3. Performance of MTD for a code with $R = 1/2$ in a channel with AWGN

where constants c_1 and c_2 are small integers [8, 9, 10].

The fact, that during decoding each iteration in the MTD actually requires only a number of simplest operations of the addition and checking types, results in the situation when the iterations number growth gives almost no decrease of the actual performance rate of the decoder, for example, in the software realization variant. As a demonstration of this algorithm property we can refer to a demo program of MTD for a convolutional code with $R = 3/4$. . It is used in the system of specialized digital television and ensures performance of about 4...10 Mbit/s, on an ordinary personal computer, which ensures with ample reserve the processing of a color television signal with a very low signal/noise ratio. The codes used in this case are subjected to the standardization procedure. The demo program and a simple manual to it can be taken from a large topic website of the SRI RAS www.mtdbest.iki.rssi.ru, where it is displayed in the education section. Actually the comparison of MTD with other methods demonstrated that the processing rate on the MTD basis appears to be only about by two orders of magnitude higher than with the use of, for example, turbo decoders with comparable coding gain [17]. A demo program for a LDPC code is also to be found there.

In case of hardware realization of MTD, for example, at FPGA Xilinx or Altera, the tests carried out confirmed their good efficiency parameters with simultaneously very high throughput up to 1,6 Gbit/s. Such an opportunity emerged after the realization of patented engineering solutions for hardware MTD. Pursuant to these solutions such a decoder turns into a single-cycle decision making circuit, and within each cycle it is able to take up to 40 decisions on the symbols under decoding, while the limiting frequency of cycles is determined by the maximum possible rate of the shift of data received from the channel by the decoder's shift registers, which mostly comprise it. Typical rate of the information progress along the shift registers stated in the FPGA is within the range of 100-250 Mbit/s, and the number of functioning in MTD in parallel registers of the type may exceed a hundred. That means, that the hardware realization performance can formally considerably exceed even 10 Gbit/s. In fact it removes all the restrictions on the processing speed for such devices, which, with the ensured by the multithreshold algorithms efficiency energy parameters, makes them the sole leaders among all the other methods of digital streams transmission via the costly satellite and other channels. In particular, the already developed hardware MTD versions for earth remote sensing systems are especially useful, because it is their high-speed flows of digital data with limited transmitter power that should be protected in every possible way using the error-correcting coding methods. An image of a quick model of an MTD decoder at FPGA Altera is given in Fig. 4.

Figure 4. A model of an MTD decoder at FPGA Altera

9. THE USE OF MULTITHRESHOLD DECODERS IN PARALLEL CODING CIRCUITS

For the approximation of the boundaries of the MTD efficient performance to the channel capacity it is possible to use it in circuits with parallel concatenation [9, 10, 18]. The basis of these circuits' architecture comprises the singling out in the self-orthogonal code C_0 with a code distance d_0 and code rate R_0 of a certain constituent code C_1 with the code rate $R_1 > R_0$, which is also self-orthogonal. The code distance d_1 of the singled out code is chosen so as to be much less than d_0, and, as follows from Fig. 3, its efficient performance area will be closer to Shannon's bound. While decoding a parallel code several decoding iterations of the constituent code C_1, are realized, which enables to lower by about an order the BER in the information sequence received from the channel; after that the remaining part of code C_0 is introduced into the decoding process. A distinctive feature of this parallel concatenation circuit is the fact that the external code is functioning at the code rate of R_0, while in the customary concatenation codes the external code's code rate is close to 1. This feature ensures a considerable advantage of the MTD over other concatenation structures.

Thus, for example, in Fig. 5 the results of a simulation for circuits with parallel coding in a channel with AWGN for a self-orthogonal code with $R_0 = 6/12$, $d_0 = 13$ and $R_0 = 5/10$, $d_0 = 15$ (curves ≪Parallel≫) are given. In this case, in the parallel code with $d_0 = 13$ an external code with $R_1 = 6/11$, $d_1 = 7$, was singled out, and in the code with $d_0 = 15$ a code with $R_1 = 5/9$, $d_1 = 9$ was singled out. The curves ≪Constituent≫ in the figures reflect BER performance at the output of the singled out codes of the parallel circuit. Dot lines without markers in these figures are used to show BER of the optimum decoding of codes with $d = 7$, 9, 13 and 15. For comparison sake parameters of the decoded with the help of MTD customary self-orthogonal codes with similar d and R (curves ≪Customary≫) are also shown in Fig. 5. Let it be noted that for obtaining the parameters' values rather short codes were used, with the length of several thousand bits, and 10. . . 20 decoding iterations. With a certain increase of the decoder memory volume and a number of the iterations realized via parallel concatenation we have already managed to get the parameters represented in Fig. 5 by the ≪Long≫ curve. As follows from the analysis of the given graphs the use of parallel coding enables MTD to function with a little over 1,5 dB of the channel capacity.

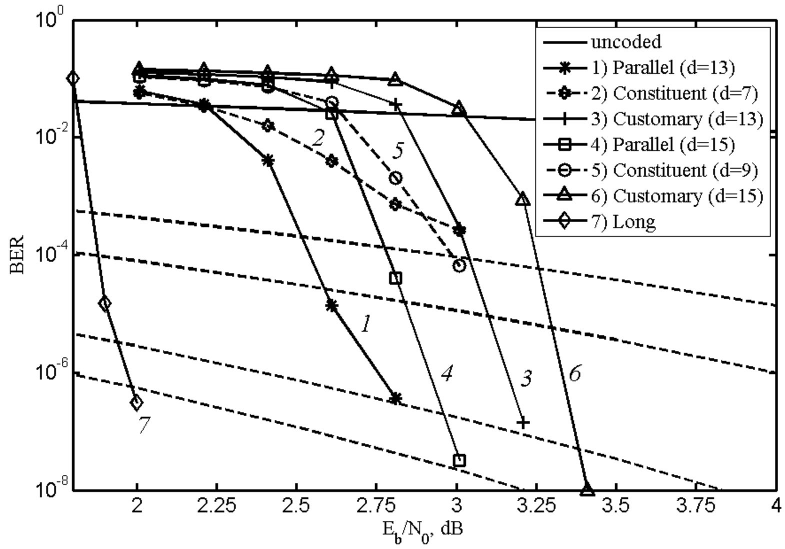

Figure 5. Simulation results for a parallel code on the basis of MTD in a channel with AWGN

The MTD complexity during parallel coding (in the terms of the number of operations performed) appears even less than the complexity of a customary MTD, as in this case during the first decoding iterations certain elements of the syndrome register simply do not participate in the sum calculation process at the threshold element.

10. CONCATENATED CODES DECODED WITH THE USE OF MTD

The high MTD performance parameters contribute to its wide use as a component of different coding structures as their efficiency is directly connected with the efficiency of their constituent elements.

A special place among the code circuits on the basis of MTD is taken by its concatenation with parity check codes, the use of which enables to boost the efficiency of the coding application. The specific nature of the following circuit amounts to the concatenation requiring practically no extra equipment expenditure (for example, only one modulo-two adder should be introduced into the coding circuit), while the use in a concatenated code, for example, Reed-Solomon codes, appears to be much more difficult. The principles of MTD concatenation with a parity check code are considered in [9, 10, 18].

The efficiency of performance of concatenated circuits comprising a self-orthogonal code with $d = 7$ and 9 and parity check code with the length of 50, for a channel with AWGN is given in

Fig. 6. As can be seen from the figures, the concatenated code on the MTD basis turns out to be much better than an unconcatenated one. At that the use of the simplest parity check code together with a self-orthogonal code enables to obtain an additional coding gain of about 1. . . 1,5 dB with an BER at the decoder output amounting 10^{-5}. It should also be noted that a concatenated code comprising a Reed-Solomon code (255, 223, 33) and a convolutional code with a code rate of 1/2 and code length restriction $K = 7$, decoded with the help of an optimum Viterbi algorithm, even with a lower total code rate ($R \approx 0,437$) appears inferior to a concatenated circuit on the MTD basis with BER grater than 10^{-6}. It should be mentioned that using concatenation with a parity check code together with the considered previously parallel coding might result in obtaining additional coding gain. An example of performance of such a circuit is given in Fig. 6 via curve ≪MTDp+PCC≫. The use of a low redundant code in the external cascade of the circuit enables to get as small as desired error probability with energy loss of about 0,1. . . 0,2 dB.

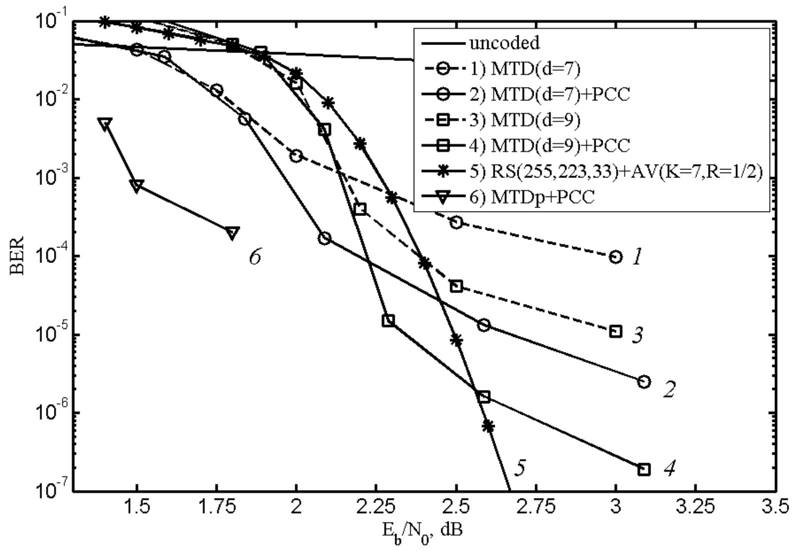

Figure 6. Simulation results for a concatenated code on the MTD basis in a channel with AWGN

The complexity of the decoding of the analyzed concatenated circuit as compared with the complexity of a customary MTD is increased for the complexity of the parity check code decoder amounting to approximately only two operations per information bit.

11. MULTITHRESHOLD ALGORITHMS IN NON-EQUAL ENERGY CHANNELS

Let us describe a representative example of a very simple modification of the MTD algorithm, which, taking into account the concrete method properties, is making very precise use of its properties, and practically without any complication of the method itself improves its energy efficiency performance crucially in total accordance with the principle ≪from simple to efficient≫. It is just that the correlation of the signal system with the properties of the code and decoder is taking place here.

Let us consider a two-channel scheme of the transmission of digital data via satellite, space and other communications channels with rather a high level of Gaussian noise. Let us choose for a certain signal/noise ratio, initially the same for each of the considered communications channels, such a redistribution of the total integral energy that should be able to ensure a maximum possible independent subsequent coding of the received information symbols on the basis of multithreshold decoding of binary block or convolutional codes. In other words the minimum level of the realization of the error propagation effect during majority decoding should be chosen as a criterion of the best energy redistribution among the channels. These issues have been rather well investigated in the MTD theory [6, 9]. The reduction of this effect enables to achieve a considerable increase in the convergence of the MTD decisions to optimum ones, which creates conditions for a more efficient performance of the MTD algorithm under high noise levels.

Different ways of energy balancing can be considered for the forming of such an absolutely simple new signal-code construction. For example, channels may be organized in such a way that the code information symbols are transmitted via one of them and check codes via the other one. In this case the error propagation analysis is maximum simplified, which enables to carry out rather prompt and complete consideration of applicability of a maximum number of codes and their respective MTD algorithms in such coding circuits. Such models got the name of non-equal energy channels [9, 19]. They can be easily realized in ordinary digital data transmission channels.

As detailed analysis of a number of codes and certain MTD algorithms' modifications for non-equal energy channels with different parameters has demonstrated, the transfer of the MTD efficient performance area boundary towards a higher noise level of the channel in the code rate R range from 1/4 to 3/4 might amount to 1 dB, which is of utmost importance, as even the initial MTD efficiency in ordinary channels appears to be rather high. At that the channels' energy ratio should be within the range from $1,3$ to $3,2$.

The necessity of functioning under higher noise level conditions requires an increase of the number of decoding iterations in the MTD, but such an increase usually turns out to be not greater than twofold, which enables to maintain the low complexity of the MTD realization both in the hardware and in the software formats.

The new results obtained in this field are substantiated by graphs in Fig. 7, which demonstrate the potential of the introduced algorithms and the already known methods. The graph ≪1) MTD-X≫ corresponds to the efficiency of a MTD decoder at FPGA Xilinx, curves ≪5) MTD+CC1≫, ≪6) MTD+CC2≫ and ≪7) MTD+CC3≫ are given for the MTD application in the simplest concatenated circuits with parity check control. All these have been discussed in great detail in [15]. Fig. 7 also gives efficiency curves for a Viterbi algorithm with a standard code with the length $K = 7$ (curve ≪3) AV, K=7≫) for a concatenated circuit of a Viterbi decoder with a Reed-Solomon code (curve ≪4) AV+RS≫), for a turbo code (curve ≪9) Turbo≫) [2] and a low-density parity-check code recommended in the DVB-S2 standard [20] (curve ≪10) LDPC≫). It should be noted that for the realization of a high speed decoder of low-density parity-check codes at FPGA the efficiency loss amounts to more than 0,5 dB (curve ≪11) LDPC (FPGA)≫). The ≪$C = 1/2$≫ vertical characterizes the capasity of a Gaussian channel, the designers keep striving at obtaining improving the decoding characteristics at the code rate of $R = 1/2$. ≪MTD-L≫ – is a long code and a MTD decoder with $I = 40$ iterations realized in the SRI RAS at FPGA Altera. The new result for a MTD and a non-equal energy channel is represented by a dot line ≪8) MTD+NEC≫. It means an opportunity of a very simple and considerable increase of the code decoding efficiency under a delay in decision taking not exceeding 400000 bits, under which the well known and rather high MTD functioning rate is preserved both in the software and (especially) in the hardware format.

Taking into account the achieved closeness of the MTD efficient functioning area to the communications channel's capasity, the prospects of MTD for further approximation of its characteristics to the Shannon bound can be considered to be good. At that, a sufficient advantage of MTD over other algorithms in the number of operations amounting to one-two decimal exponents for different combinations of coding parameters, gives ground to assume that it is possible to use MTD

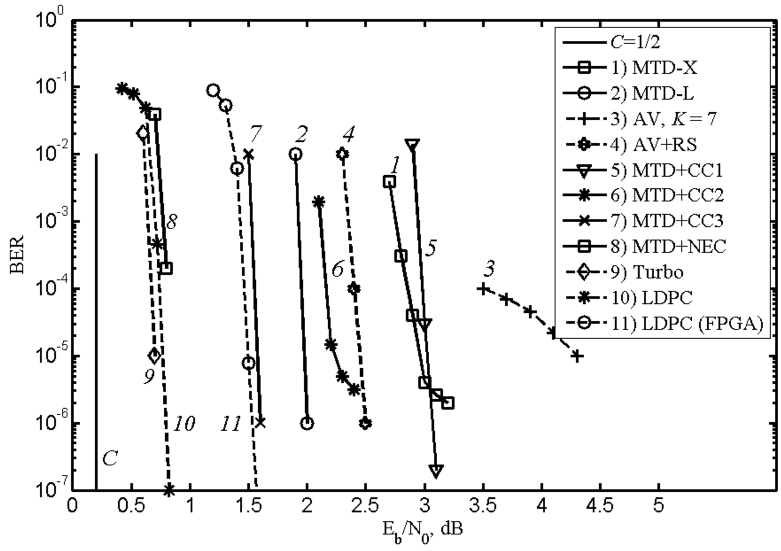

Figure 7. Performance of MTD, Viterbi decoder, turbo and low-density parity-check codes in a Gaussian channel with $R = 1/2$

actively in the development of advanced digital data transmission hardware for space and satellite communications channels.

12. CONCLUSION

The use of MTD in satellite and other costly channels enables to realize randomly high processing rate and boost their coefficient of performance considerably. An extremely simple arrangement of MTD as compared with other methods efficiency comparable with it makes them preferential for hardware realization in quick broadband channels. In rather low speed communications channels even software MTD realizations are most efficient; they require the composition of only several dozen commands of the program code for the threshold element. Simple methods of codes and signals correlation boost the MTD potential even further and make its realization especially easy. The absolutely insignificant difference in the efficiency of MTD and certain most complex decoders of other types, will be, apparently, overcome in the near future as follows from the dynamics of the MTD characteristics improvement.

Thus, as a result of an almost four decades research effort, a broad class of multithreshold algorithms has been developed, which might appear of use for many modern high speed communications systems with maximum possible coding gain levels and unattainable for other algorithms processing speed.

The MTD research testifies to the effect that the algorithms with irrational use of computational resources are by far behind the much easier methods, which solve the decoding challenge more efficiently and economically. It is apparent that the issue of the complexity of the coding realization will continue to exist in the foreseeable future, and due to the growth of information exchange speeds and the necessity of increasing the integrity of information during its transmission and storage, the demand for a prompt decoder realization will be all the more urgent. Especially preferable among all the realization variants will be algorithms performing only very easy, uniform and speedy operations. MTD conform to these requirements to the utmost. And the compliance of its potential with the characteristics of the most complex algorithms makes multithreshold algorithms all the more attractive.

More detailed information and research results on MTD are given at the specialized website of the SRI RAS www.mtdbest.iki.rssi.ru.

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