Optimum Decoding Performance on the Basis of Multithreshold Algorithms

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Abstract: *The iterative majority improved decoders are described. They are called multithreshold decoders (MTD). These decoders have a property of convergence to the solution of the optimum decoder with keeping linear complexity of implementation, which one is a property of usual threshold procedures. Different decoding applications are discussed. Experimental results are submitted. Decoder for non-binary MTD considered also.*

Keywords: iterative process, optimal decoding, majority and multithreshold decoding, MTD, parallel concatenation, nonequal channel energy, QMTD, non-binary majority decoder.

1. Introduction

Theoretical and application results about multithreshold decoding (MTD) algorithms for binary digital streams in Gaussian channels with a large noise level were considered in [1,2,3,7]. They are development of classic threshold decoder [11]. Some new important MTD properties are discussed. The essentially improved error correction algorithms of MTD type in channels with high noise level are considered also. Useful new channels with unequal energy are described. Code parameters and decoder performance are discussed. Possible MTD characteristics in concatenated schemes will be submitted also. Performance for non-binary MTD called QMTD is discussed.

2. Main MTD Properties

In accordance with the key MTD algorithm property all decoding symbols changes always lead strictly to optimum decoder (OD) decisions if error correction continues [1,3,7,9,10]. Any analogues of such significant properties for other error correcting algorithms till now are not present.

Classes of codes were found for MTD which are not subject almost to effect of error propagation (EP), i.e. groupings of errors at the output of the threshold decoder. All earlier used approaches to studying EP effect could not give anything constructive for idea of repeated error correction.

MTD decoder actually reaches the OD decision in many cases at rather high noise levels. At the same time, though achievement of optimum decoder decisions usually demands total search methods, complexity of algorithm MTD grows with code length only linearly.

At our specified bilingual web-site **www.mtdbest.iki.rssi.ru** it is possible to find the detailed developed answers to questions asked frequently by the readers, wishing to improve their understanding error correcting coding problems, including MTD decoding.

3. Parallel Code Concatenation

MTD decoders are especially convenient and effective for parallel codes. Effective parallel MTD coding schemes perhaps have appeared much earlier than all others similar methods at all [2,10]. Well known now idea of parallel coding applied to MTD decoding becomes simultaneously simpler in realization and more effective in error correction for large noise level. Let it be any binary self-orthogonal code (SOC) marked as $\mathbf{C_0}$ with code rate $\mathbf{R_0}$. Let they distribute check symbols in such a way that one of two parts of check array is more large than second part. We may consider this case as appearance two parallel codes with essentially different code rates. Then if at first step decoder works with code $\mathbf{C_1}$ and $\mathbf{R_1} \gtrsim \mathbf{R_0}$, then at second step instead second code $\mathbf{C_2}$ with $\mathbf{R_2} \lesssim 1$ it is possible to decoding full concatenated code with rate $\mathbf{R_0}$ as a whole code. This possibility to decode at second step code $\mathbf{C_0}$ with low rate $\mathbf{R_0}$ instead high rate code $\mathbf{C_2}$ is a very useful chance. The parallel concatenation has possibility to use codes $\mathbf{C}_2$ and $\mathbf{C}_1$ with different minimal distances and other parameters. Another useful convenience appears in fact that MTD during decoding code with parallel concatenation must only change check sets, which are used in majority decoding. Such a flexibility MTD algorithms for parallel concatenation creates possibilities for different improvements in code efficiency.

Substantially for this reason MTD for concatenated codes are especially effective, remaining to be simple, as well as usual MTD algorithms.

4. Non-Equal Energy Channels

Let's consider the two-channel circuit of the Space or satellite channels with large enough level of Gaussian noise. We shall choose for some signal/noise ratio, originally identical for each of two considered channels, such a distribution of the general total energy to provide the best possible subsequent decoding the received information symbols in binary block or convolution codes [1-3,6]. Criterion of the best redistribution of energy between channels is a minimum level of error propagation effect (EP) at majority decoding. In theory MTD these questions are fully enough investigated [2]. Decrease in error propagation effect allows to improve considerably MTD decisions convergence to optimum, that, in turn, creates conditions for more effective MTD algorithms work at the large noise levels.

For such simple enough signal-code design various ways of power balancing may be considered. For example, discussing two channels can be organized in such a manner that in one of them information code symbols, and in another – check bits are transmitted.

Necessity of communication equipment working at higher noise levels demands increase in number of MTD iterations. A practice and modeling of MTD algorithms for NEC has shown that such calculation increase usually appears no more than double, that results in small complexity of MTD realization both in soft, and in hardware variants.

5. Experimental Results

The new received results in this area are illustrated by curves at Pic.1 on which opportunities of the suggested algorithms and already known methods are submitted. The curve MTD-X corresponds to efficiency of MTD decoder at PLIS Xilinx, curves MTDmd2 and MTD+CC2 are given for MTD application in the elementary concatenated circuits. All these algorithms were in details discussed in [2]. They are concatenations usual SOC with simplest parity check codes (**d**=2). Curve marked as MTD+CC3 uses external code with minimal distance **d**=5. Curves for Viterbi algorithm (VA) with a standard code of length **K**=7, for concatenated circuit VA with Read – Solomon (RS) code (VA-RS), and for a turbo code [8] also are submitted. Vertical bound **C**=1/2 defines capacity of Gaussian channel, equal **C**=0,5 to which developers aspire at improvement of decoding characteristics for **R**=1/2. Another result marked as MTD-L corresponds MTD for a long convolutional code with decision delay ~ 400’000 bits. It is useful to note that it is effective decoder without any concatenation and with enormous throughput that may be realized on the basis PLIS Altera. The new result for MTD with NEC channel - line MTD-NEC - corresponds to an opportunity of simple MTD in NEC channel.

6. Complexity

Taking into account achieved MTD throughput in a communication channels, it is possible to consider, that MTD has good prospects on the further approach of its characteristics to Shannon bound. Thus significant advantage MTD before other algorithms on number of the operations, achieving one ÷ two decimal power for various combinations of coding parameters [1,2,6], gives the basis to believe, that just MTD is necessary to use actively for development the modern equipment for the Space and satellite communication channels in the case of very high channel rates.

In the channel with rather large noise level at modeling MTD work with usual personal computer its throughput is more than 1 Mbit/s per 1 GHz the processor clock frequency that exceeds extremely productivity of other soft algorithms at the same signal/noise efficiency. This result was a reason to use soft MTD decoder in special mobile digital TV system [2].

Since MTD contains many parallel registers and all calculations may be done in one step in hardware form, then decoding rates from 500 Mb/s up to 2Gb/s and even more are not very difficult tasks for this algorithm.

7. Non-Binary Multithreshold Decoders

Let it be further a linear non-binary code, which check matrix has the same view, as well as in a binary case, i. e. it consists of zeroes and ones only. Let this matrix corresponds to self-orthogonal systematic block or convolutional code (SOC) [3,7-10]. In this case code words of minimum weight **d**, where **d** - is a minimum code distance, have an alone non-zero character $\mathbf{i_k}$, with value $\mathbf{q_i}$, $\mathbf{q_i} > 0$, in its information part. As check (so, and generating also) the matrix contain only zeroes and ones, the operations of the encoder and decoder with checking characters of a syndrome **S** in the received word are only additions. Thus, coding and decoding do not need processing in non-binary fields or in rings for integers. It is only enough to arrange integer group. It essentially simplifies principally all coding procedures and subsequent decoding.

Non-binary MTD (QMTD) looks for error values, which are most frequently numbers in check sets for decoding symbols. For this QMTD the main theorem about decision convergence to OD is proved also [2,7]. Different error probabilities estimations for QMTD and experimental results are now known too [2,7,12-14].

At Pic.2 for codes with small redundancy - **R**=0,95 - characteristics QMTD and codes RS with length **n**=256 are submitted. The curve for code RS with **n**=256 and **R**=7/8 is shown also. Dotted lines M80Kb1 and M80Kb2 specify opportunities of two QMTD for codes length **n**=80000 and symbol size 1 and 2 bytes. From comparison RS codes of length **n**=256 at **R**=7/8 and **R**=19/20 with **n**=256 ÷ 4096 it is clear, that it is more difficult to provide a good efficiency for RS code redundancy reduction. Nevertheless characteristics of codes with little redundancy majority decoding on QMTD basis appear to be very good and can provide a high levels of noise immunity if the chosen codes have big enough lengths. There are many possibilities further QMTD performance improvement for all discussed codes and decoder parameters.

It is useful to note that QMTD is extremely simple. Demo program for QMTD with **R**=0,95 code (it may be rewritten from our web-site) give 4* 10^10 (10 billions!) decoded bits and even more at standard PC during one hour work for a very high channel noise probability p0~10^-2 if R=0,95 in the case of 4 byte decoding symbols (compare with Pic.2).

QMTD is essentially useful for coding very large data bases with extremely high reliability and controlled integrity. Codes of Reed-Solomon class will never achieve these performance at all.

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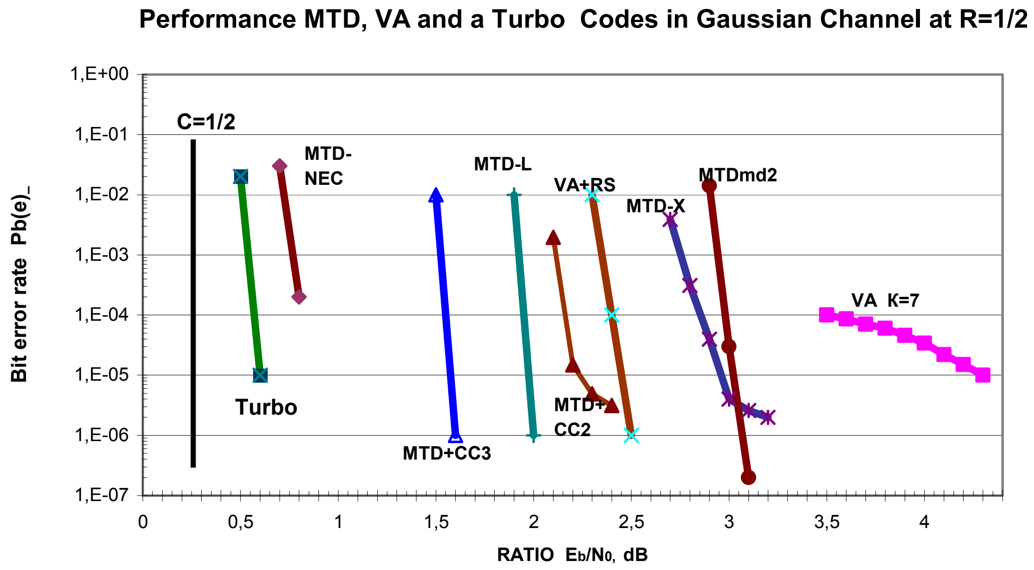
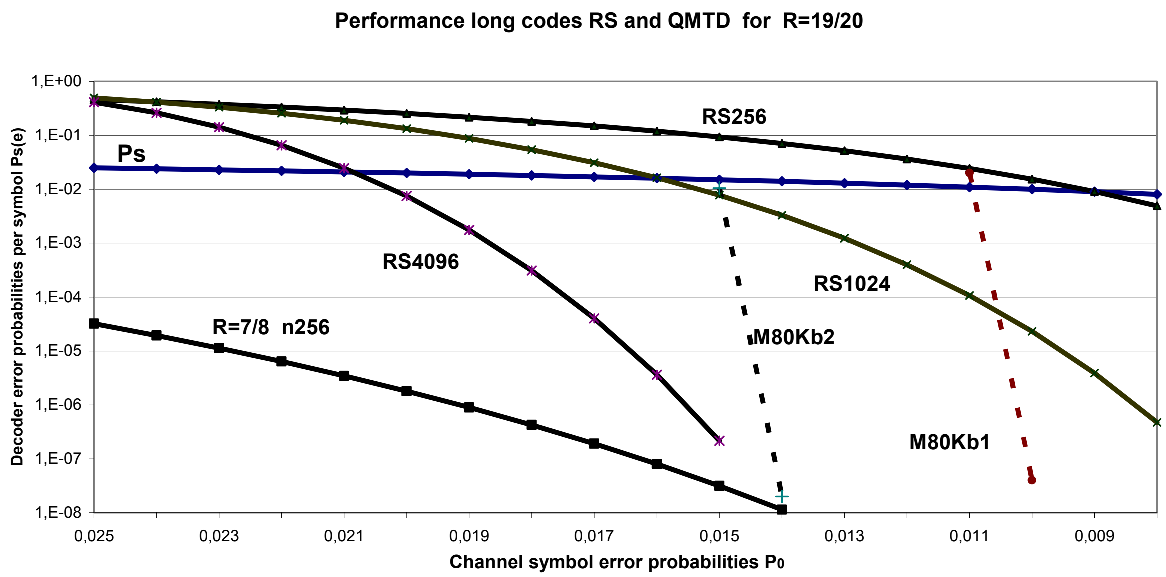


Picture 1.



Picture 2