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Non-Binary Multithreshold Decoders with Almost Optimum Performance

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Abstract: *The simplest majority type decoders for the character data decoding are described. They are called non-binary multithreshold decoders (QMTD). These decoders have a property of convergence to the solution of the optimum decoder with keeping linear complexity of implementation, which one is property of threshold procedures. Decoder error probabilities are discussed. Experimental results are submitted also.*

Keywords: iterative procedure, optimal decoding, majority and multithreshold decoding, MTD.

1. Introduction

The multithreshold decoders (MTD) are further development of a customary majority algorithms [7] and provide decoding, in many cases simply conterminous with optimum methods, that correspond to total search in efficiency. Here we shall consider generalization of multithreshold decoding (MTD) for the binary data in Gaussian channels [1,2,4-6,9] at non-binary symmetrical channels. The value of this method is a result of the fact, that the majority algorithms have only linear growth of complexity (decoding operation number) with code length **n** increase. As far as usually optimum methods are characterized with exponential rising complexity with code length, the usage of non-binary MTD, described further as QMTD, seems specially desirable. It is more important, that in case of large values of a code basis q, q > 10, it is practically impossible to create truly effective optimum decoders (OD), as in this case their complexity in most cases will look like $\mathbf{q}^{\mathbf{k}}$, where **k** - is the length of the encoding register. It also determines the relevance of application QMTD usage, as far as the capabilities of decoders for Read - Solomon (RS) codes are limited very much.

2. Non-Binary Multithreshold Decoder

Let consider usual **q**-ary, **q** > 2, symmetrical channel (QSC) with an error probability $\mathbf{p}_s > 0$, when a transmission any initial character of a code transforms it to one of stayed (**q**-1) characters incidentally, separately and with equiprobability. For such a channel the optimum decoder solution will be such, probably, unique code word among $\mathbf{q}^{\mathbf{nR}}$ possible ones, which word differs from the received word in a minimum number of code characters. (Here it was supposed, that **n** - code length expressed by a number of a code characters, **R** - code rate, **R=k/n** < 1.)

Let it be further a linear non-binary code, which check matrix has the same view, as well as in a binary case, i. e. it consists of zeroes and ones. Let this matrix corresponds to self-orthogonal systematic block or convolutional code (SOC) [3,7-10]. In this case code words of minimum weight **d**, where **d** - is a minimum code distance, have an alone non-zero character $\mathbf{i}_\mathbf{k}$, with value $\mathbf{q}_\mathbf{i}$, $\mathbf{q}_\mathbf{i} > 0$, in its information part. As check (so, and generating also) the matrix contain only zeroes and ones, the operations of the encoder and decoder with checking characters of a code formation and calculation of a syndrome **S** in the received word are only addings. Thus, coding and decoding do not need processing in non-binary fields or in rings for integers. It is only enough to arrange integer group. It essentially simplifies principally all coding procedures and subsequent decoding.

Let's assume, that QMTD decoder is arranged so, that after customary syndrome vector **S** calculation for received code word the main decoding procedure is doing. Decoder contains additional differential register **D** also, which one has length **k** and is equal **0** at start moment.

At main step for an every next controlled information character $\mathbf{i}_\mathbf{k}$ decoder scores number and defines two most often meeting values of checks, relating it, for example, **q**1 and **q**2, but **q**1 meets **m**1 times, **q**2 - **m**2 times, **m**1 > **m**2, and the remaining values of checks for a decoding character $\mathbf{i}_\mathbf{k}$ meet nor more than **m2** times. In this case decoder changes decoding symbol $\mathbf{i}_\mathbf{k}$ **,** all checks and corresponding symbol $\mathbf{d}_\mathbf{k}$ of vector **D** at value **q2 .**

It is clear, that if for two most often meeting checks values equality **m**1=**m**2 is true, the character $\mathbf{i}_\mathbf{k}$ does not change and the decoding attempt for any other information character of a code may be done.

Then we choose a new decoding symbol and so on. Process may by stopped after **N** iterations of decoding or if no any symbol changes occurred during next iteration.

Note that this QMTD does only operations of adding, subtraction and comparison for example in modulo **q** group. So the decoder is extremely simple. This generalization for threshold decoding was made in 1984 [1-3,5,6].

3. Main QMTD Decoding Theorem

For QMTD algorithm main theorem about growth its decision verisimilar was proved [1,3]. At each change of a character $\mathbf{i_k}$ QMTD decisions converges to more verisimilar decisions. The most essential circumstance increasing correcting abilities of described non-binary QMTD, is the capability to make the error-free solutions at large values **q** even with only 2 right checks for $\mathbf{i_k}$ among **d** possible checks. It usually occurs in the case, when incorrect checks $\mathbf{s_i}$ concerning decoding character $\mathbf{i_k}$ have all different values $\mathbf{s_i}$, $\mathbf{s_i > 0}$.

If first try to correct received symbols diminish. information bit error probability, then it is useful to realize next decoding iterations. Estimations for first symbol decoding error probabilities [1,3,6] have shown that QMTD begins really to improve its decisions for more large input error probabilities in **q**-ary channel (**q**>>1) than in a binary case. For example, for **R**=1/2 in binary symmetric channel (BSC) Massey's threshold decoder (and binary MTD also) improve their decisions for channel error probability $\mathbf{p_0}$~0,06 or less. But in QMTD for **q**=256 at first iteration real error probability improvement is possible if $\mathbf{p_s}$~0,22.

This QMTD properties provide very good potential possibilities if it will be real to construct conditions for fast successive convergence MTD decisions to optimal decoder (OD) results [2,3].

4. Optimal q-ary Decoder Effectiveness

Let's consider, how it is possible to compute the lower estimation of optimum decoding probability for a code assigned in a described above way. In all cases it will be detection of most often met conditions that the error vector will have its Hamming distance to the nearest minimal weight code word smaller, than an error vector Hamming weight. Due to code linearity it is reason for the incorrect symbol decision even in the case of optimum algorithm with total search. Esteeming such error vector, we should allow, that it is necessary to analyze only those positions of error vector, which ones correspond to checks concerning the current decoding character $\mathbf{i_k}$.

Such error vectors are following [1,5,6]:

- All check symbols and decoding character $\mathbf{i_s}$ are erroneous:
  $P_1(e)=p_0^d$,
  where **d**=**J**+1, **d** - minimal code distance in SOC, J – number of code orthogonal checks ;
- All check symbols are erroneous, but two of them are identical, and $\mathbf{i_o}$ is received correctly:

$$P_2(e)=(1-p_0)J(J-1)p_0^J \prod_{i=1}^{J-2}(1-i/(q-1))/(q-1)/2;$$

- There are one correctly received check symbol, and others are erroneous, as well as $\mathbf{i_s}$:
  $P_3(e)=J(1-p_0)p_0^J$.

Events listed above are quite enough for majority of actual conditions for codes usage to receive preliminary satisfactory estimations of a potential noise immunity of a code. And as QMTD at each step converges to the OD solution, it is possible to expect, that at some high noise level this MTD in most cases will reach the optimum result with minimum decoder error probability.

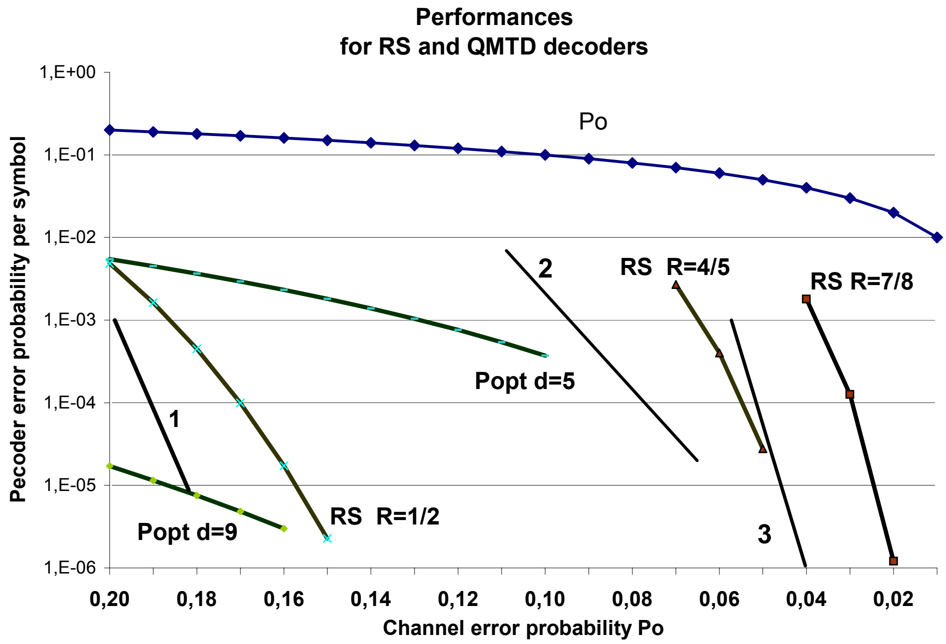
Additional situations in channel error distribution leading to OD errors are considered in [1,3,5,6].

5. Simulation Results

It is especially convenient in technical systems to deal with the byte data structure. Let's remember, that except for RS codes there are no other some effective relatively simple decoding methods for the non-binary (character) data. But codes RS with simple decoding are too short. For QMTD there are no any limitations in code length at all, as it works simply in a group of integers, dealing only with operations of adding and matching in selected set. So dealing with long codes in QMTD they can get results similarly binary almost optimum decoding for long and very long codes with least complexity.

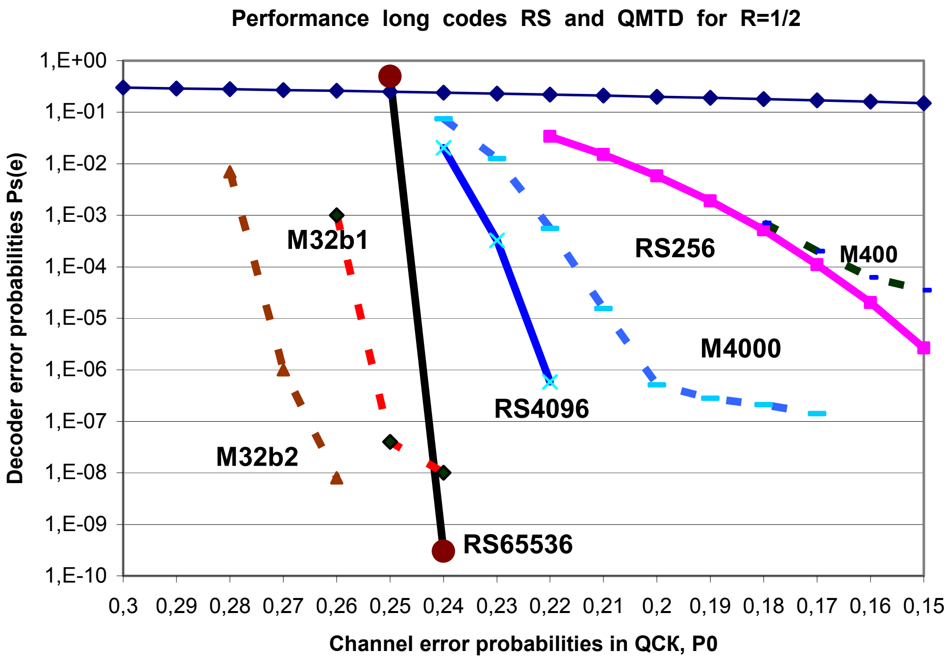
The characteristics of decoders for RS-codes and QMTD in a non-binary symmetrical channel with independent accidental errors (QCK, by analogy with customary binary BSC) are at Pic.1 submitted. For achievement the solution usually conterminous with optimum or close to the OD solution, it is necessary 5÷20 decoding iterations in QMTD for the received block. It completely corresponds to MTD method for binary codes [2-5].

A view of the curves for mean symbol error decoding probability $\mathbf{P_s(e)}$ is shown at Pic.1 as function of a channel QCK probability $\mathbf{p_0}$ at the input of RS-code decoders for code rates R=1/2, **R**=4/5 and **R**=7/8, when **n**=255. A very simple QMTD for **q**=256 provides much



Pic.1

higher characteristics, than for a RS-code usual decoder, due to a greater length **n**=1000 and good convergence QMTD's solutions to the OD decision (lines 1, 2 and 3 for simplest QMTD with the same code rates: 1/2, 4/5, 7/8).

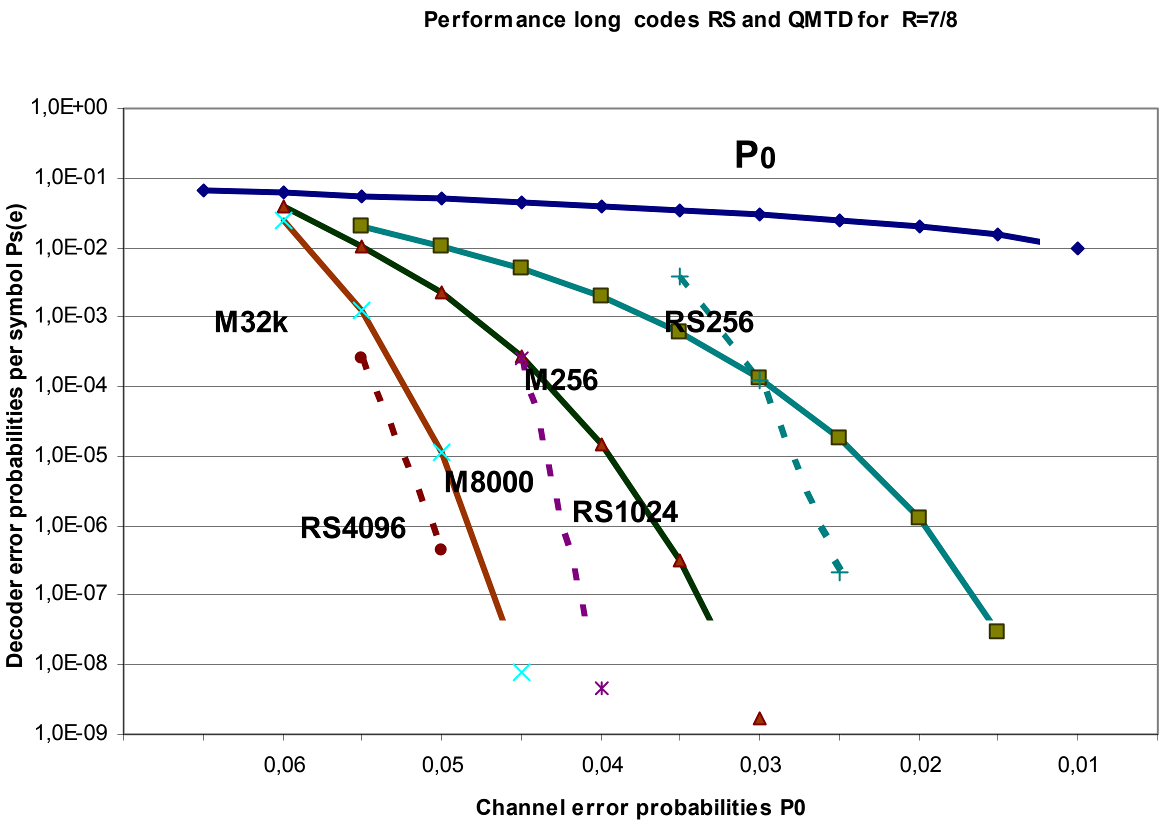


Pic. 2

There are low bound P_{opt} for OD and self-orthogonal codes, when **d**=5 and **d**=9 at the Pic.1 also.

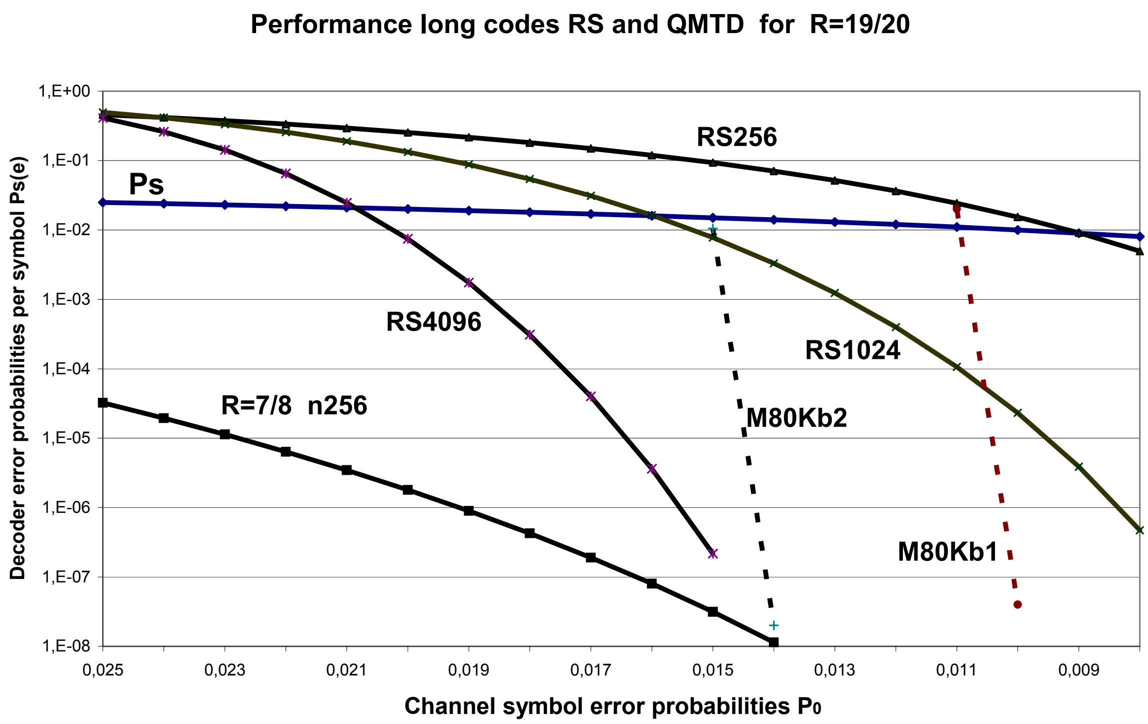
At increase code length the characteristics of QMTD can be essentially improved without any additional computational complexity.

Pic.2 shows simulation results for long codes with **R**=1/2. The symbol error probabilities for Read - Solomon codes are submitted at this picture, which are designated as RS**n** where **n** - the code length expressed as



Performance long codes RS and QMTD for R=7/8
1,0E+00
1,0E-01
1,0E-02
1,0E-03
1,0E-04
1,0E-05
1,0E-06
1,0E-07
1,0E-08
1,0E-09
Decoder error probabilities per symbol Ps(e)
P0
M32k
RS256
M256
M8000
RS1024
RS4096
0,06
0,05
0,04
0,03
0,02
0,01
Channel error probabilities P0

Pic.3.



Performance long codes RS and QMTD for R=19/20
1,E+00
1,E-01
1,E-02
1,E-03
1,E-04
1,E-05
1,E-06
1,E-07
1,E-08
Decoder error probabilities per symbol Ps(e)
RS256
Ps
RS4096
RS1024
R=7/8 n256
M80Kb2
M80Kb1
0,025
0,023
0,021
0,019
0,017
0,015
0,013
0,011
0,009
Channel symbol error probabilities P0

Pic.4

number of code symbols. Codes RS with 4096 symbols length and, especially, with **n**=65536 (each symbol - the 16 bits size), in the foreseeable future will not be subject to realization.

Here dashed lines show opportunities of codes with majority QMTD decoding at **R**=1/2 for a case **q**=256 (every symbol - one byte) for different lengths of self-orthogonal codes (SOC), which ones are the same codes, as in binary case. For QMTD it is possible to build long SOC codes with arbitrary values of code distance **d** and code rate **R**. These codes are marked as M400 and M4000 with the numbers designating lengths of codes, expressed by number of symbols. Further, designation M32b1 corresponds QMTD for a code length **n**=32000 and one-byte symbols. They can see at Pic..2 that QMTD opportunities in all cases are comparable or they are better, than for rather complex standard decoders for codes RS. Moreover, very simple for realization MTD decoder for SOC code of length **n**=32000 appears capable to provide with the elementary majority methods a noise immunity essentially unattainable even for code RS of length **n**=65536 (with two-byte symbols), the decoder for which will not be created never. Performance QMTD for SOC code with **n**=32000 is signed as M32b2 for two-byte symbols. This QMTD practically is as simple as one-byte decoder. The usual microprocessors quickly work with one-byte symbols, and with 2 and even sometimes with 8-byte words. So QMTD will be always simple.

Further at Pic.3 opportunities QMTD and codes RS are shown at code rate **R**=7/8. Continuous lines in the same designations, as at Pic.2, submit decoder error probabilities in symbols for codes RS.

Dashed lines mark SOC codes with QMTD decoding and lengths from **n**=2560 to **n**=32000 bytes (one symbol - 8 bits): M2560, M8000 and M32к. Similarly to case **R**=1/2, the opportunity of codes RS creation for length **n**=4096 at **R**=7/8 in the near future remain very problematic while even for codes of length **n**=32000 bytes considered non-binary QMTD will stay very simple.

At last, at Pic.3 for codes with small redundancy for **R**=0,95 similar characteristics QMTD and codes RS are submitted. For comparison on Pic.3 the curve for code RS with **n**=256 and **R**=7/8 from Pic.2 is resulted also. Dotted lines M80Kb1 and M80Kb2 specify opportunities of two QMTD for codes length **n**=80000 and symbol size 1 and 2 bytes.

From comparison RS codes of length **n**=256 at **R**=7/8 and **R**=19/20 it is clear, that it is more difficult to provide a good efficiency for redundancy reduction. Nevertheless characteristics of codes with little redundancy majority decoding on QMTD basis appear very good and can provide a high levels of noise immunity if the chosen codes have big enough lengths.

There are many possibilities further QMTD performance improvement for all discussed codes and decoder parameters.

6. Decoder Complexity

Let's emphasize also, that, according to the general principles of the coding theory, usageconcatenation coding methods will improve QMTD characteristics even more. Resulting decoder complexity will increase in comparison with initial algorithm very insignificantly. In details complexity QMTD was considered in [1].

Real very little calculations number in QMTD decoder which is carrying out only adding and comparison operations, can be easily proved by its soft realizations. Simulating program of this algorithm for the personal computer with middling processor speed for sets of typical parameters of a code and a **q**-ary channel can decode about one billion symbols (i. e. ~ 10^10 bits!) per one hour. Such a demo program fulfills total modeling functions of the coder, noise channel simulator and actual QMTD decoders , considered above.

The further improvement decoding efficiency is possible at transition to convolutional codes, methods of parallel coding, application of codes with the allocated branches and to other ways, the part from which was described in [1-4,6,10].

Two examples of these quick soft QMTD decoders everybody can find, rewrite to their computer and test. These demo programs are placed at educational page of SRI RAS web-site: www.mtdbest.iki.rssi.ru .

6. New non-Binary Code Applications

In addition to natural using described simple highly effective coding methods in communication networks it is necessary to show new good opportunities for QMTD applications. This method for information coding can be used for coding CD and DVD disks and other carriers great volumes of the information in accordance with future new standards. QMTD may be used in the superbig bases of audio and video data, with much higher reliability level, than it was accessible until recently, and at updating, restoration and use the stored data also. Thus it is easy to provide and the operative constant control over quality of the stored information, also data updating and arising defects of the memory carrier.

Essentially new level of a noise immunity achievable with QMTD, allows to solve the listed problems without new variants of MTD algorithms or only at their little adaptation to requirements of new scale digital systems.

7. Conclusions

The opportunity of very simple error correction in long non-binary codes at the efficiency close to a level, accessible only for optimum total searching algorithms, opens essentially new opportunities for coding the symbolical information, i. e. the main kinds of the data practically uniquely used by a modern information society. Coding provides high controllable quality of the stored, transmitted and formed information. Application of very simple and simultaneously highly effective coding methods can create new high standards of a supply with information of all aspects of civilization development.

So, complexity of new QMTD decoders is a very low and performance is very high.

Most of special algorithms, more effective than standard methods for RS codes, appear too difficult in realization for concrete systems and long codes. Sometimes similar new developed RS decoders have low effectiveness improvement for R>1/2, i. e. in the field which was discussed here above. This circumstance allows to consider that QMTD algorithms may easily find their applications in wide technical spheres.

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The additional information about QMTD - at specialized thematic bilingual web-site SRI of the Russian Academy of Sciences **www.mtdbest.iki.rssi.ru ,**

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