Optimum Decoding Characteristics Achievement on the Basis of Multithreshold Algorithms

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Abstract: *The iterative majority improved decoders are described. They are called multithreshold decoders (MTD). These decoders have a property of convergence to the solution of the optimum decoder with keeping linear complexity of implementation, which one is a property of usual threshold procedures. Different decoding applications are discussed. Experimental results are submitted. Decoder realization at the PLIS basis is discussed also.*
Keywords: iterative process, optimal decoding, majority and multithreshold decoding, MTD, parallel concatenation, unequal channel energy, .

1. Introduction

Theoretical and application results about multithreshold decoding (MTD) algorithms for binary digital streams in Gaussian channels with a large noise level was considered earlier at 7ISCTA'03 [1].

In present report some new important MTD properties are discussed. The essentially improved error correction algorithms of MTD type in channels with very high noise level are considered also. Useful new channels with unequal energy are described. Code parameters and decoder performance are discussed. Possible MTD characteristics in concatenated schemes will be submitted also.

2. Main MTD Properties

In accordance with key MTD algorithm property all decoding symbols changes always lead strictly to optimum decoder (OD) decisions if error correction continues. Any analogues of such significant properties for other error correcting algorithms till now are not present.

Classes of codes were found for MTD which are not subject almost to effect of error propagation (EP), i.e. groupings of errors at the output of the threshold decoder. All earlier used approaches to studying EP effect could not give anything constructive for idea of repeated error correction.

MTD decoder actually reaches the OD decision in many cases at rather high noise levels. At the same time, though achievement of optimum decoder decisions usually demands total search methods, complexity of algorithm MTD grows with code length only linearly.

At our specified bilingual web-site **www.mtdbest.iki.rssi.ru** it is possible to find the detailed developed answers to questions asked frequently by the readers, wishing to improve their understanding error correcting coding problems, including MTD decoding.

3. MTD for Erasure Channels

In channels with erasures MTD works almost at the capacity of such a channel, at many decimal exponents reducing a remained number of the erased symbols in comparison with their initial density in input digital stream. It seems to be practically unattainable for other methods also. Restoring erased data for MTD is even easier task, than for decoder in binary symmetric channel (BSC), though complexity of MTD for errors is very insignificant too. For example for erasure channel probability $\mathbf{p_0}$~0,4 and short code with rate **R**=1/2 a very simple MTD can diminish resulting part of non-restored symbols to the level ~0,002. If the code length will be increased and concatenation may be used part of remaining erasures will be less then 10^{-6} with minimum calculations.

4. Data Compression

MTD is possible to apply for simultaneous error correcting and data compression, in particular, with binomial source statistics. It is very important, that for some types of sources compression MTD may be realized at the efficiancy level very close to theoretically limiting possibilities. And, that is very essential, such MTD is not afraid even high error density in the accepted compressed streams. In this case it restores the data with required high quality also. There are not finding out any attributes of «fragility» of the compressed information in restored flow at all when distortions in the transmitted data lead usually to the big packets of error in the received data.

An example of MTD used for channel coding and data compression may be the next [2]. Let it is code with rate **R**=**k**/(**k**+1), **k**=2,3 … . If they will consider that information transmitting through channel is zero, then decoder parameters will not be changed. Next, if errors in information flows becomes new information and check symbols of code save there primary sense, then it appears that only check bits transmission is necessary to determine information "ones" in information bits. Check bits are in this situation in fact syndrome vector also. So after check bits transmission and subsequent decoding they get information flow with length, which is in **k** times more long, than length of check part of discussing code. So code with R>1/2 always can be used for certain data compression in noisy channels. Such description MTD application with high decoding parameters allow to use this algorithm in more wide signal processing domain .

5. Parallel Code Concatenation

MTD decoders are especially convenient and effective for parallel codes. Effective parallel MTD coding schemes perhaps have appeared much earlier than all others similar methods at all [2,10]. Well known now idea of parallel coding applied to MTD decoding becomes simultaneously simpler in realization and more effective in error correction for large noise level. Let it be any binary self-orthogonal code (SOC) marked as $\mathbf{C_0}$ with code rate $\mathbf{R_0}$. Let they distribute check symbols in such a way that one of two parts of check array is more large than second part. We may consider this case as appearance two parallel codes with essentially different code rates. Then if at first step decoder works with code $\mathbf{C_1}$ and $\mathbf{R_1} \gtrsim \mathbf{R_0}$, then at second step instead second code $\mathbf{C_2}$ with $\mathbf{R_2} \lesssim 1$ it is possible to decoding full concatenated code with rate $\mathbf{R_0}$ as a whole code. Just possibility to decode at second step code $\mathbf{C_0}$ with low rate $\mathbf{R_0}$ instead high rate code $\mathbf{C_2}$ is a very useful chance. Decoding code with parallel concatenation has possibility to use codes $\mathbf{C_2}$ and $\mathbf{C_1}$ with different minimal distances and other parameters. Another useful convenience appears in fact that MTD during decoding code with parallel concatenation must only change check sets, which are used in majority decoding. Such a flexibility MTD algorithms for parallel concatenation creates possibilities for different improvements in code efficiency.

Substantially for this reason MTD for concatenated codes are especially effective, remaining thus almost so simple, as well as usual MTD algorithms.

6. Non-Equal Energy Channels

Let's consider the two-channel circuit of Space or satellite channels with large enough level Gaussian noise. We shall choose for some signal/noise ratio, originally identical for each of two considered channels, such a distribution of the general total energy to provide the best possible subsequent decoding the received information symbols in binary block or convolution codes [1-3,6]. Criterion of the best redistribution of energy between channels is a minimum level of error propagation effect (EP) at majority decoding. In theory MTD these questions are fully enough investigated [2]. Decrease in error propagation effect allows to improve considerably MTD decisions convergence to optimum, that, in turn, creates conditions for more effective MTD algorithms work at the large noise levels.

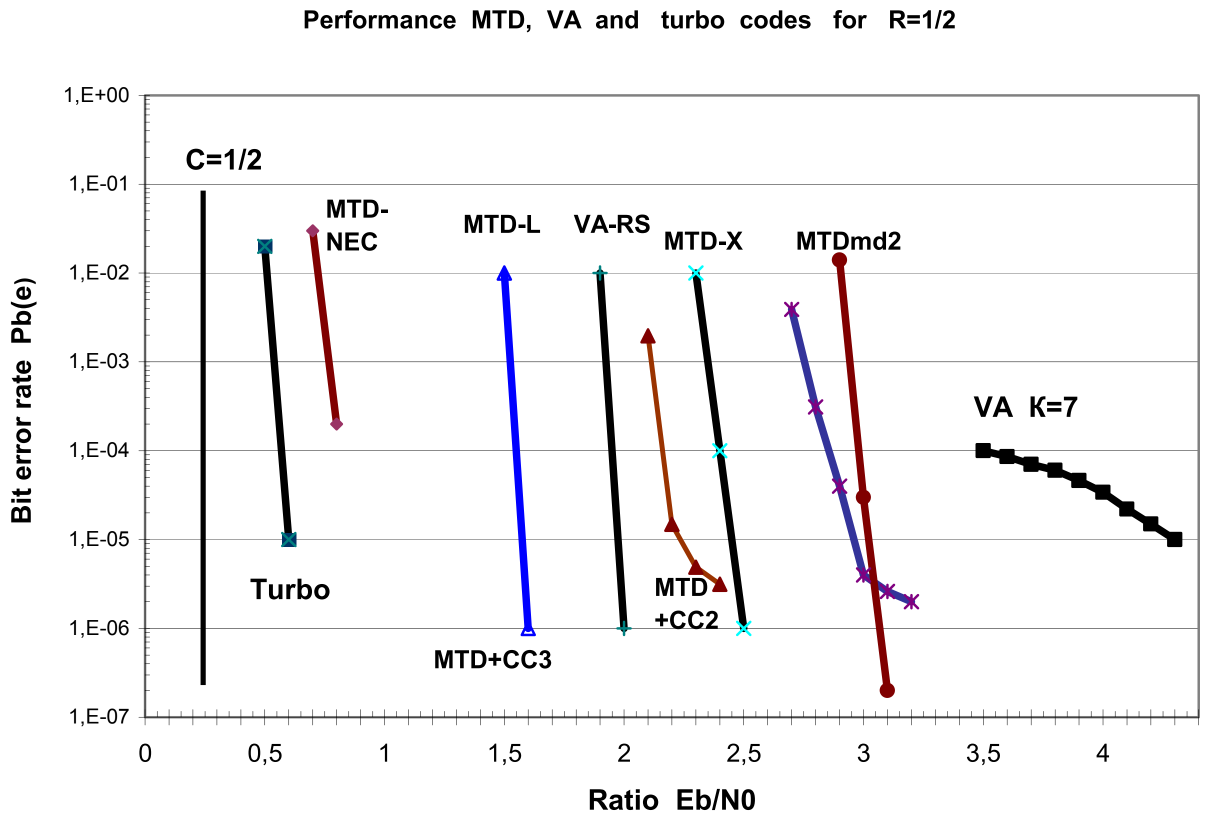
For such simple enough signal-code design various ways of power balancing may be considered. For example, discussing two channels can be organized in such a manner that in one of them information code symbols, and in another – check bits are transmitted. In this case analysis EP becomes simpler, that allows to consider easily applicability of the maximum number of codes and corresponding to them MTD algorithms in similar coding circuits. Such models of channels were named with non-equal power channels (NEC) [2,5]. They can be simply realized in usual parallel channel groups.

As the detailed analysis of some codes and MTD algorithms for channels with various parameters and non-equal power has shown, domain of effective MTD decoder work moves to higher channel noise level in a range of code rates $\mathbf{R}$=1/4 ÷ 3/4. The bound of effective MTD work can was moved to more noisy level up to 1 dB, that is very important, since initial efficiency MTD in channels of usual type appears already rather high [1-4,6]. The channels energy ratio must be within the limits $\mathbf{a}$=1,3 ÷ 3,2.

Necessity of communication equipment working at higher noise levels demands increase in number of MTD iterations. A practice and modeling of MTD algorithms for NEC has shown that such calculation increase usually appears no more than double, that results in small complexity of MTD realization both in soft, and in hardware variants.

7. Experimental Results

The new received results in this area are illustrated by curves at Pic.1 on which opportunities of the suggested algorithms and already known methods are submitted. The curve MTD-X corresponds to efficiency of MTD decoder at PLIS Xilinx, curves MTDmd2 and MTD+CC2 are given for MTD application in the elementary concatenated circuits. All these algorithms were in details discussed in [2]. They are concatenations usual SOC with simplest parity check codes ($\mathbf{d}$=2). Curve marked as MTD+CC3 uses external code with minimal distance $\mathbf{d}$=5. Curves for Viterbi algorithm (VA) with a standard code of length $\mathbf{K}$=7, for concatenated circuit VA with Read – Solomon (RS) code (VA-RS), and for a turbo code [8] also are submitted. Vertical bound $\mathbf{C}$=1/2 defines capacity of Gaussian channel, equal $\mathbf{C}$=0,5 to which developers aspire at improvement of decoding characteristics for **R**=1/2. Another result marked as MTD-L corresponds



Pic.1

MTD for a long convolutional code with decision delay ~ 400’000 bits. It is useful to note that it is effective decoder without any concatenation and with enormous throughput that may be realized on the basis PLIS Altera. The new result for MTD with NEC channel - dotted line MTD-NEC - corresponds to an opportunity of simple MTD in NEC channel with delay no more than 200’000 bits.

The specified substantial improvement of efficiency of multithreshold algorithms approximately at 1 dB in comparison with others MTD decoders submitted at Pic.1, was achieved during period from the moment of submitting last our report. With the account of already achieved MTD throughput in a communication channels, it is possible to count, that MTD has good prospects on the further approach of its characteristics to Shannon bound. Thus significant advantage MTD before other algorithms on number of the operations, achieving one ÷ two decimal power for various combinations of coding parameters [1,4,6], gives the basis to believe, that just MTD is necessary to use actively for development the modern equipment for Space and satellite communication channels. Planned achievement $\mathbf{E_b/N_0}$~0,5 dB for **R**=1/2 seems to be real in the 2008 beginning. Corresponding MTD decoder will save its very large throughput in Gaussian channels.

8. Decoder Throughput

Results of hardware realization MTD on basis PLIS Xilinx and Altera show an opportunity of achievement an absolute degree of parallel calculations. In this case the decoder architecture can be built in such a manner that the decoder as though does not spend for operations with the data of the syndrome registers in general any time at all. Directly at the moment of shift data cycle end through registers of the decoder the decision about error value in decoding symbols always are already ready. Therefore throughput of MTD algorithm can be determined only by the greatest possible speed of simultaneous data shift in all such memory registers of which ones almost completely this decoder will consist. This opportunity is already successfully realized on standard PLIS due to creation of corresponding adaptive majority element circuit and selection necessary differentials in generating code polynomials. Such two stage parallelism in MTD operations forms its super fast work.

Another way to increase decoding speed is to arrange MTD decoders in such a way that it will have many parallel shift register for information and check flows. Number of such synchronized registers may be 4 ÷ 100. Since every shift register may have time clock rate more than 50 ÷ 200 Mbit/s , achieving arbitrary large decoding throughput in MTD becomes a simple task.

Thus, advantage of hard MTD before other algorithms at speed at 2÷3 decimal exponents is already achieved also.

9. Decoder Parameter Exchange

Opportunities MTD on the coordinated mutual exchange between values of its parameters are very wide: memories, decision delays, number of operations, productivity, code length, redundancy, channel noise level and a code gain. It is always possible to choose such parameters of the decoder, that under any practically reasonable technical requirements for its development it will be real to create desired MTD device. For example, for the maximal speed achievement it is possible to realize high-efficiency convolutional MTD due to increase in decoder memory and decision delay. If it is necessary to decrease in a decision delay then they must to apply block codes that reduces throughput of the decoder, and at a choice of small transmission speed it is possible to load successfully the decoder with more powerful algorithm of correction with more comprehensive threshold element which will allow to work at the greater noise level, etc.

In the case of need in additional estimations efficiency of codes and algorithm MTD it is possible to address to our reference book on codes, at specialized web-site SRI of the Russian Academy of Sciences **www.mtdbest.iki.rssi.ru** or to imitate that circuits which causes questions, at the computer.

10. Complexity

Estimations of complexity of program realization show MTD advantage before other methods at ~2 decimal exponents in number of operations per bit at comparable efficiency. It is a very rare case in a history of digital processing methods development. They should be used correctly. In the channel with rather large noise level at modeling MTD work with usual personal computer its throughput is more than 1 Mbit/s per 1 GHz the processor clock frequency that exceeds extremely productivity of other soft algorithms at the same signal/noise efficiency. This result was a reason to use soft MTD decoder in special mobile digital TV system [2].

For the specialized microprocessors speed of MTD decoding can be increased considerably. Therefore it is improbable, that any other effective enough methods can be simplified in the same degree.

The report is accompanied by demonstration computer movies about multithreshold decoding, illustrating all steps of MTD work in convenient and pictorial form.

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