THE QUICK ALMOST OPTIMAL MULTITHRESHOLD DECODERS FOR NOISY GAUSSIAN CHANNELS

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Abstract. The principles of operation and performance of multithreshold decoders (MTD) are reviewed in the field of high levels of a channel noise. These methods in many cases are so effective, as optimum decoding procedures with total search. The complexity of their implementation is practically linear on code length. The generalizations of MTD for non-binary signals is suggested also.

Keywords: iterative process, optimal decoding, majority and multithreshold decoding, MTD, parallel concatenation, non-equal channel energy, QMTD, non-binary majority decoder.

1. INTRODUCTION

The advance in technology of decoding of noise proof codes within many decades surprisingly was not connected in any way to methods of the solution of a functional optimization problem for many discrete variables. Nevertheless decoding, i.e. search of the unique code word among exponentially large number of the possible messages, would be pure naturally to esteem from such stands. However, the decoding algorithms developed before have not used in any way for a search of the best decoder solutions of the well-known optimization procedures, which ones could be applied to search the code words located at minimally possible distance to the received word. But just threshold decoders (TD) [1], realizing the elementary error correcting methods, have the useful properties, which ones are indispensable for implementation valuable effective and simultaneously extremely simple optimization decoding procedures.

2. OPTIMIZATION IDEA

Let us consider an example of the simplest encoder/threshold decoding (TD) system with code rate R=1/2 and minimum code distance **d**=3, as it is shown at Fig.1.

As it follows from an appearance of the encoder and elementary decoder correcting single error, the precise copy of the encoder is created in the decoder too, which one forms estimations of code check bits with received from a channel information symbols ^***I*** of a code with some errors. These symbols appear in the decoder's point **K** and then, after addition at the half-adder with check symbols, received from a channel, ˆ***V*** will form characters of a syndrome vector **S**, which one depends only on a channel error vector. These characters also move then at threshold switch of the decoder from the syndrome register, as shown at Fig.1.

Even the shape of TD allows to find a simple way of correct optimization procedure organization, i.e. search the best possible decoder solution. Let's indicate for this purpose the fact, that has never been marked before: in the syndrome register of the decoder there is a difference on check symbols between received (with channel distortions) vector Ă and such code word Āi, with information symbols coinciding with an information part of vector Ă received from a channel.

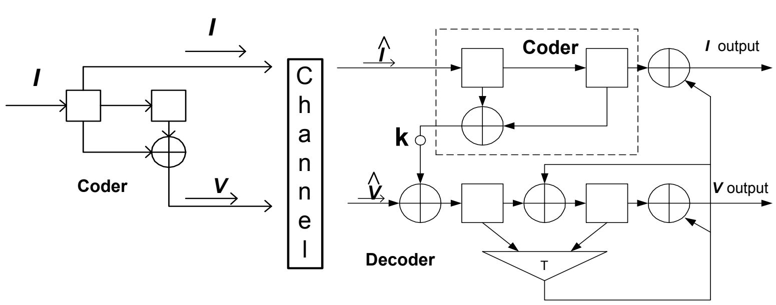

Figure 1.

It means, that the total difference between the code word - current hypothesis-solution of the decoder Āi about the sent code word and received noisy vector Ă will be in such a decoder, where in TD single vector will be added else, which one always should correspond to a difference between Ă and Āi - current decoder hypothesis on information characters, which one can change. Such decoder also will contain a current value of a full difference and, therefore, full distance between the solution of the decoder and received vector.

3. PRINCIPLES OF DECODING

This idea was realized in special multithreshold decoders (MTD) [2-7]. In accordance with the key MTD algorithm property all decoding symbols changes always lead strictly to optimum decoder (OD) decisions if error correction continues [2-4,7,9]. Any analogues of such significant properties for other error correcting algorithms till now are not present.

New classes of majority decodable codes were found for MTD which are not subjected almost to effect of error propagation (EP), i. e. groupings of errors at the output of the threshold decoder. All earlier used approaches to studying EP effect could not give anything constructive for idea of repeated error correction.

MTD decoder actually reaches the OD decisions in many cases at rather high noise levels. At the same time, though achievement of optimum decoder decisions usually demands total search methods, complexity of algorithm MTD grows with code length only linearly.

4. PARALLEL CODE CONCATENATION

MTD decoders are especially convenient and effective for parallel codes. Effective parallel MTD coding schemes perhaps have appeared much earlier than all others similar methods at all [4,8-10]. Well known now idea of parallel coding applied to MTD decoding becomes simultaneously simpler in realization and more effective in error correction for large noise level. Let it be any binary self-orthogonal code (SOC) marked as $\mathbf{C_0}$ with code rate $\mathbf{R_0}$. Let they distribute check symbols in such a way that one of two parts of check array is more large than second part. We may consider this case as appearance two parallel codes with essentially different code rates. Then if at first step decoder works with code $\mathbf{C_1}$ and $\mathbf{R_1} \gtrsim \mathbf{R_0}$, then at second step instead second code $\mathbf{C_2}$ with $\mathbf{R_2} \lesssim 1$ it is possible to decoding full concatenated code with rate $\mathbf{R_0}$ as a whole code. Just possibility to decode at the second step code $\mathbf{C_0}$ with low rate $\mathbf{R_0}$ instead high rate code $\mathbf{C_2}$ is a very useful chance. Decoding code with parallel concatenation has possibility to use codes $\mathbf{C_2}$ and $\mathbf{C_1}$ with different minimal distances and other parameters. Another useful convenience appears in fact that MTD during decoding code with parallel concatenation must only change check sets, which are used in majority decoding. Such a flexibility MTD algorithms for parallel concatenation creates possibilities for different improvements in code efficiency.

Substantially for this reason MTD for concatenated codes are especially effective, remaining thus almost so simple, as well as usual TD algorithms.

5. NON-EQUAL ENERGY CHANNELS

Let's consider the two-channel circuit of the Space or satellite channels with large enough level of Gaussian noise. We shall choose for some signal/noise ratio, originally identical for each of two considered channels, such a distribution of the general total energy to provide the best possible subsequent decoding the received information symbols in binary block or convolution codes [9]. Criterion of the best redistribution of energy between channels is a minimum level of error propagation effect (EP) at majority decoding. In theory MTD these questions are fully enough investigated [4]. Decrease in error propagation effect allows to improve considerably MTD decisions convergence to optimum, that, in turn, creates conditions for more effective MTD algorithms work at the large noise levels.

For such simple enough signal-code design various ways of power balancing may be considered. For example, discussing two channels can be organized in such a manner that in one of them information code symbols, and in another one – check bits are transmitted. In this case analysis EP becomes simpler, that allows to consider easily applicability of the maximum number of codes and corresponding to them MTD algorithms in similar coding circuits. Such models of channels were named as non-equal power channels (NEC). They can be simply realized in usual parallel channel groups.

As the detailed analysis of some codes and MTD algorithms for channels with various parameters and non-equal power has shown, domain of effective MTD decoder work moves to higher channel noise level in a range of code rates $\mathbf{R}=1/4 \div 3/4$. The bound of effective MTD work can was moved to more noisy level up to 1 dB, that is very important, since initial efficiency MTD in channels of usual type appears already rather high [4,9]. Necessity of communication equipment working at higher noise levels demands increase in number of MTD iterations. A practice and modeling of MTD algorithms for NEC has shown that such calculation increase usually appears no more than

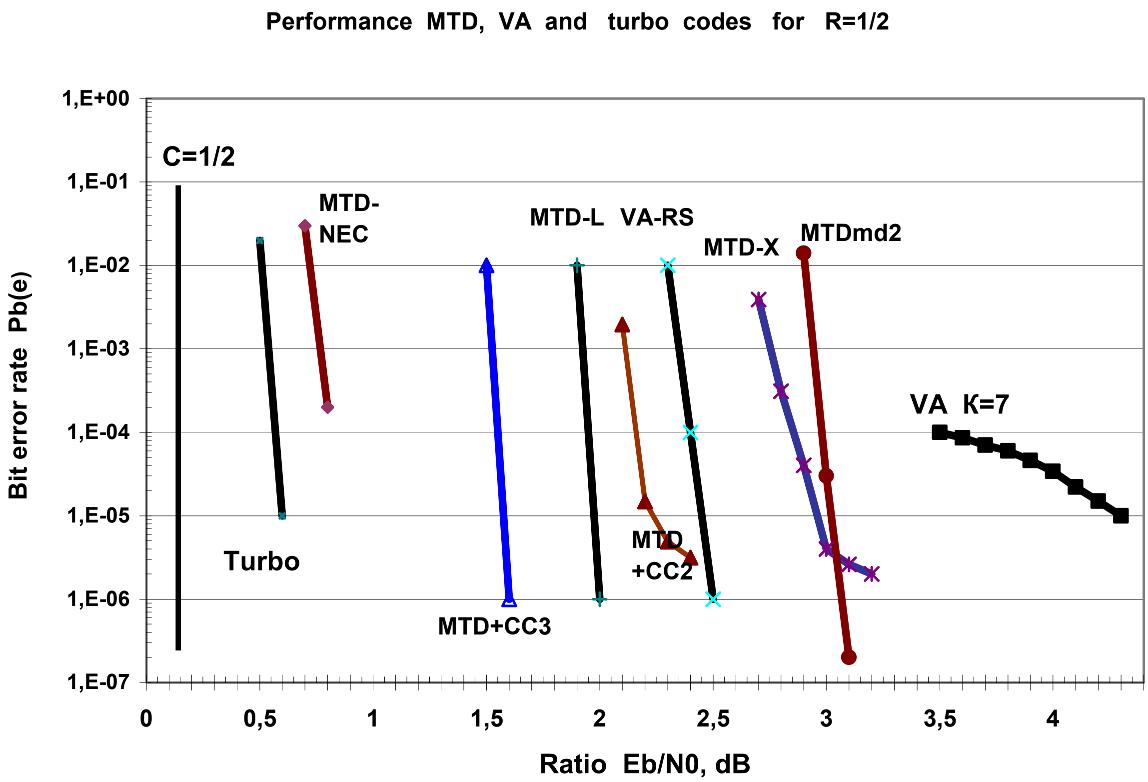

Figure 2.

double, that results in small complexity of MTD realization both in soft, and in hardware variants.

6. EXPERIMENTAL RESULTS

The new received results in this area are illustrated by curves at Fig.2 where opportunities of the suggested algorithms and already known methods are submitted. The curve MTD-X corresponds to efficiency of MTD decoder at PLIS Xilinx, curves MTDmd2 and MTD+CC2 are given for MTD application in the elementary concatenated circuits. All these algorithms were discussed in details in [4,11]. They are concatenation of usual SOC and simplest parity check codes (**d**=2). Curve marked as MTD+CC3 uses external code with minimal distance **d**=5. Curves for Viterbi algorithm (VA) with a standard code of length **K**=7, for concatenated circuit VA with Read – Solomon (RS) code (VA-RS), and for a turbo code [12] also are submitted. Vertical bound **C**=1/2 defines capacity of Gaussian channel, equal **C**=0,5 to which developers aspire at improvement of decoding characteristics for **R**=1/2. Another result marked as MTD-L corresponds MTD for a long convolutional code with decision delay ~ 400’000 bits. It is useful to note that it is effective decoder without any concatenation and with enormous throughput that may be realized on the basis PLIS Xilinx or Altera. The new result for MTD with NEC channel - dotted line MTD-NEC - corresponds to an opportunity of simple MTD in NEC channel with delay no more than 600’000 bits.

The specified substantial improvement of efficiency of multithreshold algorithms approximately at 1 dB in comparison with usual MTD decoders. With the account of already achieved MTD throughput in a communication channels, it is possible to consider that MTD has good prospects on the further approach of its characteristics to Shannon bound. MTD may be used for development the modern equipment for the Space and satellite high speed communication channels.

7. COMPUTATIONAL COMPLEXITY

The main MTD's advantage is the lowest complexity of decoding consisting, as well as in case of customary TD, in summation of weighted checks, matching with a threshold and further decoding symbols and checks change, if this threshold was exceeded. The number of iterations of decoding **I** in this case is no more than 50, and general MTD decoding complexity is estimated for **d**<25 as $\mathbf{N}_I$~(**d**+2)*(**I**+4). If under the same conditions the performance degradation MTD approximately on 0,1 d**B** channel energy is possible, that is usually could be admitted, the calculus in **MTD** are else more simplified: N_2~4***d**+3***I** .

Let's point out, that formally defined complexity as number of operations for MTD is approximately on 2 decimal order (~100 times!) less, than for turbo like codes with comparable energetic efficiency. It is essential, that at such estimations the especial difficulty of a certain part of operations, for example, taking the logarithm executed at decoding turbo codes, has not been discussed. Naturally, that by efforts of large number of the specialists engaging different codes, many algorithms were improved and simplified. But nevertheless they are now much more slower then MTD decoders.

In the channel with rather large noise level at modeling MTD work with usual personal computer its throughput is more than 1 Mbit/s per 1 GHz the processor clock frequency that exceeds extremely productivity of other soft algorithms at the same signal/noise efficiency. This result was a reason to use soft MTD decoders in a special mobile digital TV system [4,9].

For the specialized microprocessors speed of MTD decoding can be increased considerably. Therefore it is improbable, that any other effective enough methods can be simplified in the same degree.

8. MTD FOR ERASURE CHANNELS

In channels with erasures MTD works almost at the capacity of such channels, at many decimal exponents reducing a remained number of the erased symbols in comparison with their initial density in input digital stream. It seems to be practically unattainable for other methods also. Restoring erased data for MTD is even easier task, than for decoder in binary symmetric channel (BSC), though complexity of MTD for errors is very insignificant too[4,9]. For example for erasure channel probability p_0~0,4 and short code with rate **R**=1/2 a very simple MTD can diminish resulting part of non-restored symbols to the level ~0,002. If the code length will be increased and concatenation may be used part of remaining erasures will be less then 10^{-6} with minimum calculations.

9. DATA COMPESSION

MTD is possible to apply for simultaneous error correcting and data compression, in particular, with binomial source statistics. It is very important, that for some types of sources compression MTD may be realized at the efficiency level very close to theoretically limiting possibilities. And, that is very essential, such MTD is not afraid even high error density in the accepted compressed streams. In this case it restores the data with required high quality also. There are not finding out any attributes of «fragility» of the compressed information in restored flow at all when distortions in the transmitted data lead usually to the big packets of error in the received data.

An example of MTD used for channel coding and data compression may be the next [4,9]. Let it is code with rate **R**=**k**/(**k**+1), **k**=2,3 … . If they will consider that information transmitting through channel is zero, then decoder parameters will not be changed. Next, if errors in information flows becomes new information and check symbols of code save there primary sense, then it appears that only check bits transmission is necessary to determine information "ones" in information bits. Check bits are in this situation in fact syndrome vector also. So after check bits transmission and subsequent decoding they get information flow with length, which is in **k** times more long, than length of check part of discussing code. So code with **R**>1/2 always can be used for certain data compression in noisy channels. Such description MTD application with high decoding parameters allow to use this algorithm in more wide signal processing domain .

10. NON-BINARY QMTD

Let us consider generalization of multithreshold decoding (MTD) for the binary data in Gaussian channels [4,9,13-15] at non-binary symmetrical channels. The value of this method is a result of the fact, that the majority algorithms have only linear growth of complexity (decoding operations number) with code length **n** increase. As far as usually optimum methods are characterized with exponential rising complexity with code length, the usage of non-binary MTD, described further as QMTD, seems specially desirable. It is more important, that in case of large values of a code basis q, q>10, it is practically impossible to create truly effective optimum decoders (OD), since their complexity in most cases will look like $\mathbf{q}^{\mathbf{k}}$, where **k** - is the length of the encoding register. It also determines the relevance of application QMTD usage, as far as the capabilities of decoders for Read - Solomon (RS) codes are limited very much.

Let consider usual **q**-ary, **q**>2, symmetrical channel (QSC) with an error probability p_s>0. For such a channel the optimum decoder solution will be such, may be unique code word among $\mathbf{q}^{\mathbf{nR}}$ possible ones, which word differs from the received word in a minimum number of code characters.

Let it be further a linear non-binary code, which check matrix has the same view, as well as in a binary case, i. e. it consists of zeroes and ones. Let this matrix corresponds to self-orthogonal systematic block or convolutional code (SOC) [3,7-10]. In this case code words with minimum weight **d**, where **d** - is a minimum code distance, have an alone non-zero character $\mathbf{i}_\mathbf{k}$, with value $\mathbf{q}_\mathbf{i}$, $\mathbf{q}_\mathbf{i}$ > 0, in its information part. As check (so, and generating also) matrix contains only zeroes and ones, the operations of the encoder and decoder with checking characters of a code formation and calculation of a syndrome **S** in the received word are only addings. Thus, coding and decoding do not need processing in non-binary fields or in rings for integers. It is only enough to arrange integer group. It essentially simplifies principally all coding procedures and subsequent decoding.

Let's assume, that QMTD decoder is arranged so, that after customary syndrome vector **S** calculation for received code word the main decoding procedure is doing. Decoder contains additional differential register **D** also, which one has length **k** and is equal **0** at start moment.

At main step for an every next controlled information character $\mathbf{i_k}$ decoder scores number and defines two most often meeting values of checks, relating it, for example, **q**1 and **q**2, but **q**1 meets **m**1 times, **q**2 - **m**2 times, **m**1 > **m**2, and the remaining values of checks for a decoding character $\mathbf{i_k}$ meet nor more than **m2** times. In this case decoder changes decoding symbol $\mathbf{i_k}$ **,** all checks and corresponding symbol $\mathbf{d_k}$ of vector **D** at value **q2 .**

It is clear, that if for two most often meeting checks values equality **m**1=**m**2 is true, the character $\mathbf{i_k}$ does not change and the decoding attempt for any other information character of a code may be done. Then we choose a new decoding symbol and so on. Process may by stopped after **N** iterations of decoding or if no any symbol changes occurred during next iteration.

Note that this QMTD does only operations of adding, subtraction and comparison for example in modulo **q** group. So the decoder is extremely simple.

11. MAIN QMTD PROPERTIES

For QMTD algorithm main theorem about growth its decision verisimilar was proved [4,7] as it was done for BSC [4,9]. At each change of a character $\mathbf{i_k}$ QMTD decisions converges to more verisimilar decisions. The most essential circumstance increasing correcting abilities of described non-binary QMTD, is the capability to make the error-free solutions at large values **q** even with only 2 right checks for $\mathbf{i_k}$ among **d** possible checks. It usually occurs in the case, when incorrect checks $\mathbf{s_i}$ concerning decoding character $\mathbf{i_k}$ have all different values $\mathbf{s_i}$ **,** $\mathbf{s_i} > \mathbf{0}$.

If the first try to correct received symbols diminishes information bit error probability, then it is useful to realize next decoding iterations. Estimations for first symbol decoding error probabilities [4,9,14] have shown that QMTD begins really to improve its decisions for more large input error probabilities in **q**-ary channel (**q**>>1) than in a binary case. For example, for **R**=1/2 in binary symmetric channel (BSC) Massey's threshold decoder (and binary MTD also) improve their decisions for channel error probability $\mathbf{p_0}$~0,06 or less. But in QMTD for **q**=256 at first iteration real error probability improvement is possible if channel error probability is $\mathbf{p_s}$~0,22.

This QMTD properties provide very good potential possibilities if it will be real to construct conditions for fast successive convergence MTD decisions to optimal decoder (OD) results [4,9,15].

12. OPTIMAL Q-ARY DECODER EFFECTIVENESS

Let's consider, how it is possible to compute the lower estimation of optimum decoding probability for a code assigned in a described above way. In all cases it will be detection of most often met conditions that the error vector will have its Hamming distance to the nearest minimal weight code word smaller, than an error vector Hamming weight. Due to code linearity it is reason for the incorrect symbol decision even in the case of optimum algorithm with total search. Esteeming such error vector, we should allow, that it is necessary to analyze only those positions of error vector, which ones correspond to checks concerning the current decoding character $\mathbf{i_k}$.

Such error vectors are following [4,14]:

- All check symbols and decoding character $\mathbf{i_s}$ are erroneous;
- All check symbols are erroneous, but two of them are identical, and decoding symbol $\mathbf{i_0}$ is received correctly;
- There are one correctly received check symbol, and all others ones are erroneous.

Events listed above are quite enough for majority of actual conditions for codes usage to receive preliminary satisfactory estimations of a potential noise immunity of a code. And as QMTD at each step converges to the OD solution, it is possible to expect, that at some high noise level this MTD in most cases will reach the optimum result with minimum decoder error probability.

Additional situations in channel error distribution leading to OD errors are considered in [9,15].

13. SIMULATION RESULTS FOR QMTD

It is especially convenient in technical systems to deal with the byte data structure. Let's remember, that except for RS codes there are no other some effective relatively simple decoding methods for the non-binary (character) data. But codes RS with simple decoding are too short. For QMTD there are no any limitations in code length at all, as it works simply in a group of integers, dealing only with operations of adding and matching in selected set. So dealing with long codes in QMTD they can get results similarly binary almost optimum decoding for very long codes with least complexity.

The performance of decoders for RS codes and QMTD in a non-binary symmetrical channel with independent errors (QCK, by analogy with customary binary BSC) are submitted at Fig.3. For achievement the solution usually conterminous with optimum or close to the OD solution, it is necessary 5÷20 decoding iterations in QMTD for the received block. It completely corresponds to MTD method for binary codes [4,9].

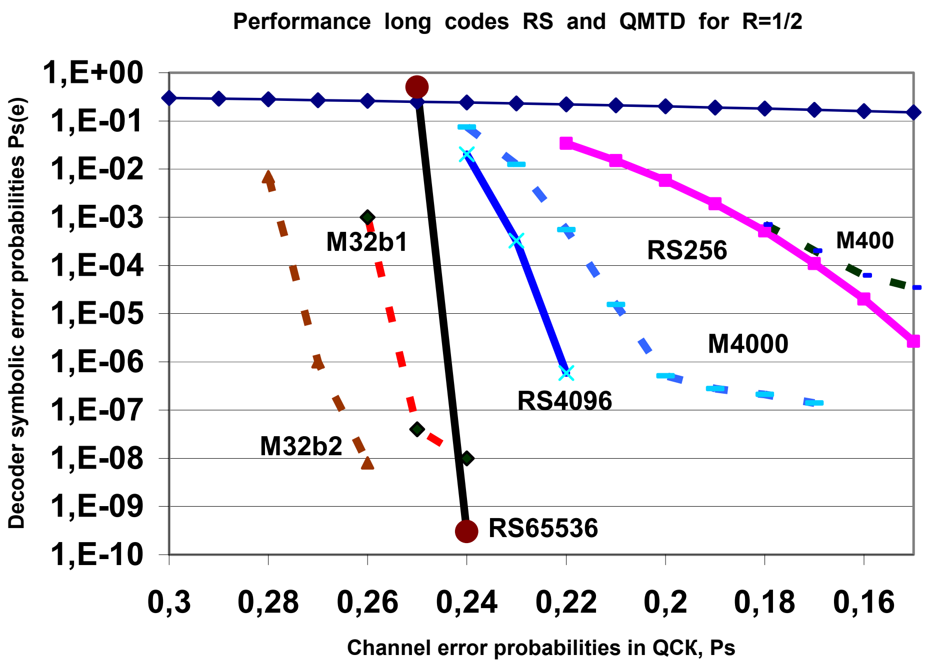

Figure 3.

Fig.3 shows simulation results for long codes with **R**=1/2. The symbol error probabilities for Read - Solomon codes are submitted at this picture, which are designated as RS**n** where **n** - the code length expressed as number of code symbols. Codes RS with 4096 symbols length and, especially, with **n**=65536 (each symbol - the 16 bits size), in the foreseeable future will not be subject for realization.

Here dashed lines show opportunities of codes with majority QMTD decoding at **R**=1/2 for a case **q**=256 (every symbol - one byte) for different lengths of self-orthogonal codes (SOC). For QMTD it is possible to build long SOC codes with arbitrary values of code distance **d** and code rate **R**. These codes are marked as M400 and M4000 with the numbers designating code length **n**, expressed by number of symbols. Further, designation M32b1 corresponds QMTD for a code length **n**=32000 and one-byte symbols too. They can see at Fig.3 that QMTD opportunities in all cases are comparable or they are better, than for rather complex standard decoders for codes RS. Moreover, very simple for realization MTD decoder for SOC code with length **n**=32000 appears capable to provide with the elementary majority methods a noise immunity essentially unattainable even for code RS of length **n**=65536 (with two-byte symbols), the decoder for which will not be created never.

Performance QMTD for SOC code with **n**=32000 is signed as M32b2 for two-byte symbols. This QMTD practically is as simple as one-byte decoder. The usual microprocessors quickly work with one-byte symbols, and with 2 and even sometimes with 8-byte words. So QMTD will be always very simple decoder.

At last, at Fig.4 for codes with small redundancy for **R**=0,95 similar characteristics QMTD and codes RS are submitted. For comparison on Fig.4 the curve for code RS with **n**=256 and **R**=7/8 is shown also. Dotted lines M80Kb1 and M80Kb2 specify opportunities of two QMTD for codes length **n**=80000 and symbol sizes 1 and 2 bytes.

From comparison RS codes of length **n**=256 at **R**=7/8 and **R**=19/20 it is clear, that it is more difficult to provide a good efficiency when the redundancy is reduced strongly. Nevertheless characteristics of codes with little redundancy majority

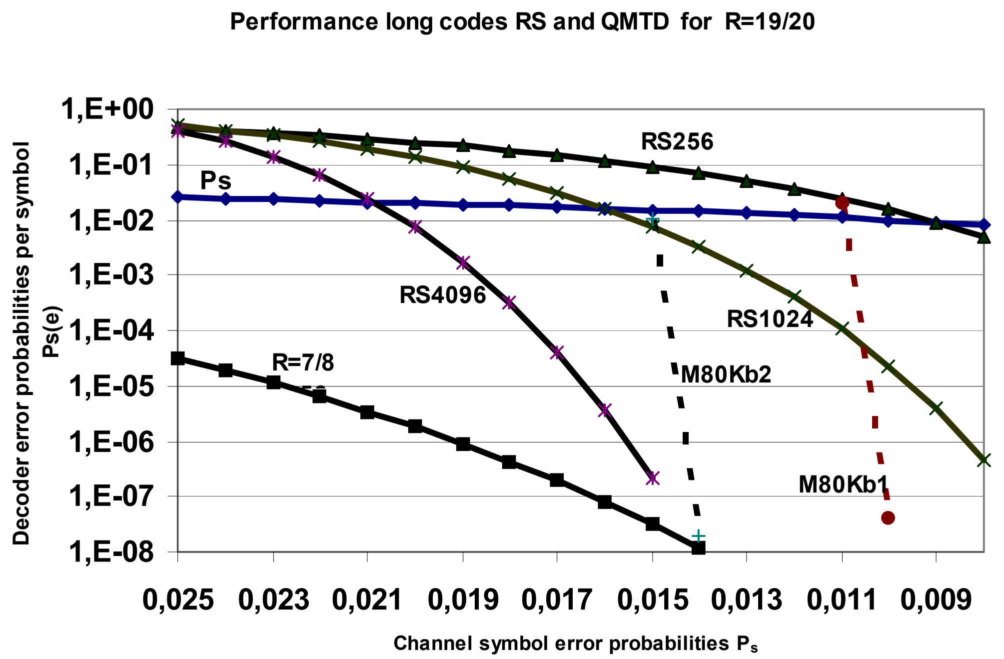

Figure 4.

decoding on QMTD basis appear very good and can provide a high levels of noise immunity if the chosen codes have large enough lengths.

14. Q-ARY DECODER COMPLEXITY

Let's emphasize also, that, according to the general principles of the coding theory, usage concatenation coding methods will improve QMTD characteristics even more. Resulting decoder complexity will increase in comparison with initial algorithm very insignificantly. In details complexity QMTD was considered in [4,9].

Real very little calculations number in QMTD decoder which is carrying out only adding and comparison operations, can be easily proved by its software realizations. Simulating program of this algorithm for the personal computer with usual processor speed for sets of typical parameters of a code and a **q**-ary channel can decode about one billion symbols (i. e. ~$3*10^{10}$ bits!) during one hour. Such a demo program fulfills in the same process total modeling functions of the information generation, coder, noise channel simulation and actual QMTD decoders, considered above.

An example of quick software QMTD decoder for R=0,95 everybody can find, rewrite to his own computer and test. All demo programs for binary and symbolic MTD are placed at educational page of SRI RAS web-site: www.mtdbest.iki.rssi.ru .

15. NEW NON-BINARY CODE APPLICATIONS

In addition to natural using described simple highly effective coding methods in communication networks it is necessary to show new good opportunities for QMTD applications. This method for information coding can be used for coding CD and DVD disks and other carriers with great volumes of the information in accordance with future new standards. QMTD may be used in the super big bases of audio and video data, with much higher reliability level, than it was accessible until recently, and at updating, restoration and use the stored data also. Thus it is easy to provide and the operative constant control over quality of the stored information, also data updating and arising defects of the memory carrier correction.

Essentially new level of a noise immunity achievable with QMTD, allows to solve the listed problems without any new variants of MTD algorithms or only at their little adaptation to requirements of new scale digital systems.

16. CONCLUSION

The binary MTD is very quick in software and hardware form. The last results submitted above show very high energetic effectiveness also. MTD is the best decoder for high speed channels. Nevertheless there possibilities will be further improved. MTD works as one step decision scheme with extremely high throughput.

The opportunities of very simple error correction in long non-binary codes at the efficiency are close to a level, accessible only for optimum total searching algorithms. QMTD opened principally new opportunities for coding the symbolical information. Coding provides high controllable quality of the stored, transmitted and formed information. Application of very simple and simultaneously highly effective coding methods can create new high standards information processing in future civilization development.

Most of special new algorithms, more effective than standard methods for RS codes, appear too difficult in realization for concrete systems and long codes.. This circumstance allows to consider that QMTD algorithms may easily find their applications in wide technical spheres.

Thus, after 30-year's researches the wide range of multithreshold algorithms are designed, which ones can be recognized by the main simple coding methods for many modern high speed communication systems with high levels of code gain and extremely large throughput for the Space, satellite channels and super large data bases.

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The additional detailed answers to questions asked frequently about error correcting coding problems, including MTD decoding, demo films about MTD, laboratory works about MTD algorithms and demo programs for quick MTD decoders with simple instructions about their application may be found at specialized large thematic bilingual web-site SRI RAS: **www.mtdbest.iki.rssi.ru** , Moscow, Russia. Every reference below (except [1,12]) may be found at our web-site also.

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