Breadboard of the submitted for publishing book V.V.Zolotarev

“ Suboptimal Multithreshold Decoders for Channels with a Large Noise Level ”

INTRODUCTION

The quick growth of volumes of data processing and the development of broad assigning digital networks presents more and more high requirements to veracity of the transmitted information. The successful activity of digital systems of communication is possible only if there is an effective instrumentation, that allows to guarantee the transmission of the discrete information with a high accuracy. The major contribution to the solution digital interchange increase problem introduces the theory of noiseproof coding realizing methods of an error control on the correcting code basis.

If the veracity increasing in feedback communication systems with error detection does not form essential difficulties, so the problem of high-performance decoding in the receiver which is not having a capabilities to organize repeated request of erroneous information (as well as in the case, when this capability is possible for the receiver seldom enough), remains rather actual.

Let's remind, that the using of coding allows to improve veracity of information exchange, to increase speed of its transferring, to reduce the size of transmitting antennas or to increase the distance, stability and reliability of communications or to lower a necessary transmitter power. The coding cooperation with other methods gives all listed above outcomes together with a transmitted data volume decrease, the information protection, narrowing of indispensable frequency band for communication and also (with?) other advantages are accessible. These merits of coding have leaded to that within several last decades the coding is a compulsory procedure of a data processing in all effective communication systems.

The necessity of usage of those or other error correcting algorithms for systems without a feedback is usually accompanied by the very rigid requirements shown to these algorithms, for example on the sizes, cost, response, energy supplying, testing, manufacturability at a hardware representation and other parameters. The large number of the regularly published monographies dedicated to different aspects of the theory of noise proof coding, testifies to complexity and urgency of this problem.

In the given book the problem’s solution of error correction in digital data on the basis of new majority decoding type algorithms is set up. The tendered and in detail considered further special multithreshold decoders (MTD) are called so because during the control of received code sequences all its information characters, which has come from a channel, are several times estimated. This correcting realized or during implementation of several attempts of symbol correcting by the set of the strictly specially arranged threshold scheme (TS), or as a result of repeated transiting of these characters by one or a group of such decision circuits.

It appears, that the procedures of a similar type at their exact implementation appear very effective. If a code and algorithm of decoding, inhering to the class of MTD methods, are selected correctly, i.e. in accordance with all requirements and limitations, which one are described in the corresponding chapters of this book, even at a noise level conforming to a case, when the code rate R only is little bit lesser, than capacity of a channel C, i.e. when it is $\mathbf{R \lesssim C}$, MTD provides high enough values of power efficiency of coding. They remember, that the measure of this efficiency is usually code gain (CG). It has large enough values, only if the decoder even at maximal with the point of view of the theory noise channel levels provides a decrease of mean error density at its output by some decimal orders with respect to error probability coming at its input. This situation really takes place for many cases of successful application MTD, when it reaches a level of decoding efficiency that is coinciding or rather close to their optimal (usually, with exhaustive exponential search) decoding.

All subsequent chapters of the book are dedicated to presentation of the problem's solution method, which one allows to reach so high performance of a multithreshold type procedures.

In the first chapter the brief resume about development of a multithreshold decoding methods is proposed. It will give the reader a definite reference points, which one will allow to the specialists interested in refinement of some concrete parts, to orient by looking up of the new publications on this problem.

Further in this chapter the basic concepts are entered and definitions used in the subsequent sections of the book the condition of applied problems of the coding theory and decoding engineering are briefly reviewed. An important point assisting in the solution of a perception aspects problem of creation and usage MTD, is the underline of steep unity of two code classes: convolutional and block ones.

The second chapter is a key for presentation of all subsequent stuff. The definitions are entered and simple auxiliary properties of codes and threshold procedures are discussed in it. It will be used then for the formulation and proof fundamental confirmation for all further presentation - the multithreshold decoder likelihood decision growth principle for linear codes. Further this MTD's possibility is extended for non-systematic and binary codes, and also for channels with erasures. The feasibility of this principle application for decoders working together with systems of multipositional signals and for some other purposes is reviewed.

In the third chapter the causes of originating of an error propagation effect (EP) are studied for threshold decoding block both convolutional binary and nonbinary codes, that has allowed to construct in essence new sets of codes, which one appear by most effective when used in MTD for a lot of channels models.

The EP effect analysis is the principle moment of research. Its comprehension makes the relevant technological fundamentals for exact formulation and subsequent solution of an absolutely special code classes construction problem, which one actually can supply the high characteristics of MTD decoding. The author does not know any papers, where the problem of such type code construction would be stated and then solved. The part of codes with the similar characteristics among a total number majority decoding codes, probably, is generally very small. It also determines the requirement for their extremely accurate construction and also simultaneous creation of the MTD class decoder, conforming to them. Though structure design procedure and optimization of parameters of MTD components is rather labor-consuming, decoder after completion of its creation process appears still actually almost so simple, as well as alone classic threshold decoder. The capabilities of the built specialized system for code construction for MTD, ensuring its high characteristic are described.

In the fourth chapter lower, upper and other approximated analytical estimations of multithreshold decoding procedures efficiency for different linear codes are injected. From a stuff of this chapter follows, that under some quite reasonable conditions and limitations on properties of codes and MTD can really have some properties, which one are generic only optimum algorithms with total search. It concerns, in particular, to an essential an error probability solution decrease the already at the second threshold scheme of MTD, and capability of almost optimum decoding in MTD, if the noise of a channel is rather small.

The fifth chapter is dedicated to the description of conditions and experimental methods of MTD investigation, obtained at simulation of this type decoder working in different channels with independent errors and erasures, and also at usage of some composite systems of signals. The characteristics of those decoders are described, which one have come in content of the communication equipment in different hardware systems.

Important point is also new special MTD description, which is capable to change the Viterbi decoder for one of standard non-systematic codes. The efficiency characteristics such MTD are close to an VA, but MTD appears much easier, and its processing speed is much higher, than for an VA.

In the sixth chapter the problems of an overall MTD performance increase are discussed at its usage in composite code designs. As it is known, the increase of coding efficiency is possible in case of new code structure construction of some base algorithms of code / decoding and procedures on the basis, which one at their proper creation provide much higher final characteristics of coding, than base algorithms, which are their components. To such approaches concern concatenation and lot of other effective methods. The features MTD and codes, used in them, allow to create rather specific code sets, which one, saving as a whole simplicity of threshold algorithms, allow follow-up to improve the characteristics MTD.

The problems of perspective MTD’s capabilities estimation are analyzed in the seventh chapter, where the versions of different decoders implementation are discussed, which ones can be even more effective.

In the Conclusion the total results on the characteristics of multithreshold algorithms and outlooks of their usage are formulated. They mark their high structural homogeneity, apparent simplicity of operation principles and bound with this property visualization of the description of their working, also their very high processing speed.

It allows to make conclusions about their high manufacturability, simplicity and low cost of implementation for many concrete communications systems.

CHAPTER 1
CODING PROBLEM IN A COMMUNICATION ENGINEERING

Brief historical help

The majority algorithm's advantage for a communication engineering consists in their extremely simple implementation and capability of correcting large number errors with weight, which is essentially more than one half of a code distance for used codes. The majority decoders were esteemed as in the classic monographies of L.Fink, L.Borodin, J.Massey, V.Kolesnik and E.Mironchkov and also in papers of many other authors.

Probably, the first attempt of theoretical perception (?) of a repeated decoding capabilities for message received from a channel with a large noise, became report of the author at a seminar on noiseproof coding in 1974. It was arranged in Scientific Council at complex problem "Cybernetics" under the auspices of Academy's of Science Presidium USSR. It was conducted by vice-Chairman of the Council professor (S.Samoilenko), where the author for the first time has submitted his results in a new research field. In the same year the characteristics of the new tipe decoder was reported on 6-th All-Union Symposium on the redundancy theory in informational systems, where these decoders were already qualified as suboptimal methods, i.e. a little differed from optimum. It was true that real improvement of a such type decoder characteristics, achived to that time, was not yet very considerable. This algorithm called as multithreshold decoding (MTD), is described also in the book "Computer networks" ("Вычислительные сети") (in the co-authorship), issued in 1981 in main Russian publishing house "Science", and in more than 80 other publications of the author.

The total number of the notes and articles on this subject already exceeds 100 titles and, surely, will increase hereinafter.

Further in the first chapter the basic concepts and definitions used in the subsequent sections of the book are entered. An important point assisting in the solution of all aspects perception MTD problem, is the consideration in this section of unity of two code classes: convolutional and block ones. The interconnection between these codes is meant in all subsequent sections of the book, where the codes dichotomy, conventional for many other publications, is not underlined at all, and the properties, received almost in all chapters, of considered algorithms and codes are implied fair both for one, and for other classes. Some cases, when it is not so, in some places of presentation are predetermined separately.

On the basis of the code gain (CG) parameter analysis which one alone also determines final value of codes validity in a communication engineering, those properties of codes are discussed also, which one they should have to be successfully decoded in conditions of a large channel noise level.

CHAPTER 2
PRINCIPLE OF THE MULTITHRESHOLD DECODER SOLUTION LIKELIHOOD GROWTH

The development of noiseproof coding methods having high performance, guesses usage of decoding procedures few distinguished from optimum or even simply optimum at least in some its most essential cases. Direct optimum decoding method (as a maximum likelihood) is very much limited because of its exponential complexity of implementation. In many cases it is proportional $\mathbf{q}^{\mathbf{k}}$, where **q** - code base, **k** - information part of the code word length **n**. Therefore it is necessary to create decoding procedures, which ones are not optimum, but at implementation are much easier then optimum, even if they have lesser efficiency. When for any algorithm efficiency difference with optimum procedures appears quite a little, such algorithm is called suboptimal. Certainly, it can appear possible only at some definite interconnections between code and channel properties, in which one the working quality of (for) this or that decoder is estimated.

In this chapter on the basis of systematic linear codes simple properties and threshold procedures they demonstrate, that in a classic binary symmetrical channel without memory (BSC), conforming to usage of “hard” modems in space, satellite and others communication channels the well constructed procedure of threshold decoding allows to correct errors some times in the received data, coded by linear block or convolutional code. Thus in all cases of changing with a threshold scheme (TS) of this or that decoding symbol the new decoder solution will be closer to the received block, i.e. more verisimilar, than previous hypothesis of the decoder about this block. Such extremely valuable property of the decoder is saved at any number of the realized acts of a code symbol correction.

It is possible briefly to call this feature of algorithm MTD as property of tendency MTD to the optimal decoder (OD) solution . So it also will be called sometimes, as it well corresponds to entity of this decoder work.

Then the characteristics of nonbinary MTD are conceded, for which one the theorem about tendency to the solution of OD in nonbinary q-ary a symmetrical channel without memory is proofed too. At sufficient number characteristics of nonbinary MTD appears much better, than the results for many Read - Solomon codes (RS codes).

They demonstrated also, that the basic properties and characteristics of MTD are saved for Gaussian channels with modems quantizing the binary solutions, i. e. for "soft" ones. Further MTD is described for non-systematic codes decoding with the same properties too. Then the possible decoder for channels with erasures is offered. For these channels the main theorem about MTD is transformed to a simple rule of effective recovery of unknown code characters.

The scheme with MTD for signal systems such as PSK-n and AFM-16 is described. It is shown, that the OD's solution the convergence principle can be spreaded and at complex with such a modem and MTD codec. For systems of this class it is possible also to expect substantial improvement of modulation characteristics and coding system, that allows not only to reduce transceiver power in a channel, but also it is simultaneously possible to reduce a bandwidth of signal transmission, as it usually takes place when multiposition modems are used.

Thus, the set of MTD's properties for a broad set of signals systems is capable to supply convergence to the optimum solution, which the achievement was considered possible earlier only for composite algorithms with total search, that for long codes will be always impossible. The admissibility of long codes usage, for which it will be realized a simple MTD procedure, creates principle conditions for implementation potentially high noise immunities with good long codes by the elementary means.

To problems of looking for conditions, at which one the process of such convergence really was so long, that MTD almost always would reach the OD solution, the following chapter about an error propagation effect for a threshold type procedures is dedicated.

THE CHAPTER 3
ERROR PROPAGATION EFFECT IN MAJORITY DECODERS

The results of the previous chapter has demonstrated a principle capability essential increase of linear codes majority decoding efficiency in all the main diversity of channels with an additive noise. At the same time, MTD is not OD for all channels examined above.

In this chapter on the basis of new treatments well-known an error propagation effect (EP) the attempt is made to understand the causes of a fast stop of change decoding characters process at repeated attempts of their correcting by usual threshold decoders.

For a considered problem of an approaching of efficiency MTD to capabilities OD most productive has appeared consideration of a EP as degrees of errors grouping in the majority type decoder. Thus it is necessary generally speaking to distinguish two completely different causes of errors grouping after decoding.

First cause is conditioned in channels without memory by selected algorithm of decoding and its features: by availability of a feedback connection (FC) from a threshold scheme to the syndrome register, decision rule for correction, way of check orthogonalization or selection of thresholds value.

The second cause appears properties of the most used code. For example, in short convolutional codes of a nonsystematic type there are usually many code words of weight approximately minimal $\mathbf{d_m}$ or free code distance $\mathbf{d_{free}}$, as results of strong errors grouping even at an output of the decoder realizing Viterbi algorithm (VA), which one is optimum for these codes.

In this chapter the special probabilistic approach to an estimation of an error propagation effect (EP) is designed. It enables as to understand influencing each of the mentioned above causes errors grouping at outputs of the threshold decoder, and to find conditions imposed on decoders and codes, with which ones they necessarily should content that the degree of grouping of errors at implementation of threshold algorithm was minimum.

For the first time applied and in detail designed for the analysis of a EP the known method of probability generator (PG) functions has allowed precisely to calculate or to give enough correct estimations for packet occurrence probabilities with two and more errors when the threshold decoder is used. Some estimation results for such probabilities for block and convolutional codes are presented in case of different modifications of the activity rules for TD, structure, codes length, and also error probability for a binary symmetrical channel (BSC).

The block and convolutional binary systematic codes of a self-orthogonal and other type, codes of maximum length, and also nonsystematic and nonbinary codes were analyzed. In addition to customary selection of checks the special schemes for some code checks orthogonalization are estimated.

The set of the obtained results has supplied (ensured?) creation of algorithms, which one for given limitations on codes and decoders parameters allow under the given initial requirements to plot the best codes with a minimum susceptibility to an error propagation effect.

It appeared, that these new codes were never really constructed specially for any decoding procedures and there are no any data about them in the coding literature. They are rather long codes, as that also the theory demands intended for work in channels with a large noise level and small resultant decoder error probability. They, as well as codes for customary threshold decoders (TD), appear easily decoded as with conventional TD, and so with the MTD help. In the latter case low levels of errors grouping even are really reached at high probabilities of an error in a channel conforming to a rather small difference between capacity of a channel **C** and code rate **R**, i.e. when it is $\mathbf{R \lesssim C}$. The conditions for implementation of repeated refinement idea for the MTD solution, in many cases really improve the solution of this decoder thereby are provided at fulfillment even of tens iterations of decoding down to achievement of the solution on a maximum of likelihood, i.e. OD solution.

Outcome of the analysis of a EP became the special applied program system for construction codes on the basis of transformations (conversions) of any initial breadboards of code polynomials, which one can be constructed within the framework of adopted limitations. The complexity of code construction was in most cases proportional to 4-th degree of a code length, that allows, basically, to decide a problem of construction practically every desired codes with demanded parameters up to code length **n** $\approx 10^5$ bits for code rate **R** = 0,1 ÷ 0,9.

Most of codes constructed on the basis of the above-stated principles and the decoders of the MTD class were designed for different hardware and software communication systems and have demonstrated those high characteristics, which were designed when developed.

CHAPTER 4
ANALYTICAL ESTIMATIONS OF MULTITHRESHOLD DECODING EFFICIENCY

The development of the noiseproof coding theory for a long time in one of the major aspects consisted in looking for more effective for its the stage of development, for example, algebraic codes and methods of their decoding. The estimation of error correcting capacity for these methods did not course any difficulties, as for this purpose, for example, binomial distributions were used.

The actual characteristics of high-performance algorithms at a large noise level they receive usually already on the basis of computer simulation. For such algorithms of decoder results strong influencing render not only code words of minimum weight, but also those ones, which weight is more, than minimum code distance **d**. Thus, the fine structure of a code, its spectrum renders decisive influencing on decoding algorithms efficiency at a large noise level of a channel. It is fair and for multithreshold decoding algorithms in a broad band of values of a channel errors probabilities and a good approaching to decoding in accordance with maximum likelihood.

Until recently after almost four decades of development of majority methods for them the probability calculus technique for the first code character erroneous decoding $P_1(e)$, suitable both for block, and for convolutional codes was known only in case of using TD with a feedback connection (FC). This probability can be interpreted and as mean probability of an error per bit P_b (e) for definite TD, for which feedback connection is missed.

In this chapter a lot of useful estimations of the MTD's characteristics is obtained. The calculation method for upper-bound estimates of mean an error per bit probability P_b (e) in block and probability P_1 (e) in convolutional MTD is offered. Despite of absolutely little as contrasted to conventional TD complicating, such decoder provides at small noise levels the characteristics a little bit distinguished from the optimum one. Certainly, for this purpose it is necessary to select good codes with respect to EP, that allows to simplify essentially suggesting estimations.

The technique permitting with the special probability generator (PG) with large dimension help to increase accuracy of estimations for the MTD probability $\mathbf{P_b (e)}$. Thus more precise estimation $\mathbf{P_b (e)}$ above can be obtained too.

Besides on the basis of the EP analysis and properties of MTD the approximated average scores of the MTD characteristics are offered, which allow to compound submission about actual borders of MTD efficiency, decoding, executing more then two attempts.

The way of calculation of the lower estimations of mean error probability per bit $\mathbf{P_b (e)}$ is indicated. The nature of errors grouping in codes used for MTD and yardstick of correspondence between the MTD and optimum decoders solutions are discussed.

The similar generalizations are fulfilled for block codes, nonbinary and nonsystematic versions of MTD implementation .

THE CHAPTER 5
THE EXPERIMENTAL CHARACTERISTICS OF MULTITHRESHOLD ALGORITHMS

The material of previous chapter confirm a capability of rather high decoding characteristics achievement on the basis of MTD algorithm without essential complicating of the initial majority type decoder. At the same time, the analytical estimations of MTD efficiency have appeared just a few distinguished with optimum for used codes at a rather small noise level and only for a case of two threshold schemes in MTD.

The indicated causes result in necessity of code characteristics analysis for decoding with large number of decoding iterations in MTD for channels with a large noise by experimental methods. It is important, that the mean probabilities of an decoding error for representative codes at a small signal-to-noise ratio will be in range $10^{-3} \div 10^{-6}$. It allows to provide a sufficient volume of computer simulation at moderate computing costs.

The obtained data testify to good conformity of experimental data and analytical estimations of capabilities MTD in a broad band of decoding error probabilities for different algorithms.

Thus, the simulation appears by an alone possible (probable) method of effectiveness MTD testing with number of iterations more than two, specially at a large noise level of a channel. It is clear, that, eventually, only such channels introduce the greatest concern at the problems solution of implementation of those or diverse decoders. Designed for the purposes of an operational analysis MTD application packages allow to model work as alone of algorithm such as MTD, and some its modifications described below. Thus the simple gang of a noise level in a Gaussian type channel, number of levels quantum on an output of the modem and borders of areas of the solutions, systems of signals of modems, number of iterations of decoding, polynomials of used codes, and also other parameters of decoders and channels is provided. The developed system of the assembly (collecting) of statistics allows to watch frame of errors MTD on separate iterations and general efficiency of activity. Besides the means of different decoder parameter adaptation to a channel and code were included in the programs of simulation. The number of such parameters can reach 1000. Usage of potent means of acclimatization on a design stage MTD allows in some cases follow-up to reduce error decoding probability by 1,5 - 2 decimal order about as contrasted to approximately good selection of basic elements of the decoder: thresholds, weights of checks and methods of their calculations, and also used code.

The similar modelling or emulating outcomes of decoders imitation programs were written and for a lot of other algorithms of decoding, in particular for algorithm Viterbi (AV) with different values of code rate **R** at length of the encoding register **Ko** < 20.

The experimental MTD characteristics analysis has demonstrated broad capabilities of these decoders in channels with independent errors or erasures of characters.

The conducted brief matching MTD with codes BCH and Read - Solomon (RS), AV and series procedures have demonstrated, that by natural comparison of these basic nonconcatenated methods MTD always appears much more effectively than algebraic procedures and AB for binary codes. In nonbinary channels it is quite comparable by the capabilities with codes RS or in some cases surpasses them, saving all advantages of simple majority errors correction circuits.

Certainly, the high characteristics MTD are accessible only with rather long codes. But their application in high-velocity satellite and space communications channels appears quite permissible. Quite opportunely also to remember, that in conditions of a large noise level the effective decoding lowering probability an error per bit $\mathbf{P_b}$ (e) at some the decimal orders, is possible only at usage not only rather long codes, but very powerful decoders, conforming to them, too. It is the immutable requirement of the coding theory. Thus, at decoding ensuring high CG values at large noise levels, any alternative to very lengthy codes and effective algorithms do not exist .

Reached with the help of enough simple MTD CG (code gain) value for channels such as BSC about 4 ÷ 6 dB and more for an error probability at the output about $\mathbf{P_b}$ (e) = 10^{-5} in many cases are inaccessible for other algorithms with reasonable complexity of implementation.

In case of using MTD with "soft" modems, as they usually do at implementation AV, they increase efficiency of the decoder by 1 ÷ 2 dB with quite permissible additional complexity of a multithreshold procedure implementation. Certainly, characteristics of "soft" MTD are unapproachable for any Viterbi decoders, as MTD even at a large enough noise level actually optimally decodes long codes, that appears always impossible in case of using AV.

The special MTD is described, intended for using instead of the AV with one of standard non-systematic codes used for AV. The efficiency characteristics such special

MTD are close to an VA, but MTD it appears much easier, and its throughout is much higher, than in case of VA for all versions of implementation of such a special decoder. This direction of researches will prolong the development.

Extending obtained the experimental outcomes, it is possible to consider, that the capabilities of relatively simple MTD appear to give code gain approximately by 2,5 ÷ 2 dB below than theoretical bounds for the corresponding codes. The ways of construction MTD are retrieved, which the characteristics will differ from these bounds less than by 1,5 dB. This is in accordance with basis of coding theory. From them it uniquely follows, that the codes lengths used for such MTD should be already rather large. There are recently designed and the perspective approaches are in detail analyzed, which the implementation one will allow closely to approach to theoretically limiting values of MTD efficiency in channels with high noise level.

CHAPTER 6
MTD USAGE IN COMPOSITE CODE SYSTEMS

The discussed above results, that concerns to the MTD characteristics, have large principle significance. For the first time for the large period of the coding theory development and decoding engineering has appeared, that one of the elementary simple among known methods of decoding - majority - can be updated in such a manner that in enough broad band of channel noise levels it differs from optimum decoders for given good codes quite a little.

The high characteristics of MTD as base algorithm provide broad capabilities for it and at its usage in different code designs, as their efficiency is directly connected with possibilities of codes, included in these constructions.

In this chapter the codes with unequal protection of characters (UPC), design of parallel codes (PC), codes with irregular energy distribution (IED) and codes with dedicated branches (CDB) are reviewed. The special place among these new code schemes takes cascading with parity check codes, which one also provide further increase of code efficiency. However cascading with such codes practically does not demand additional equipment costs, whereas usage in a cascade code, for example, Read - Solomon codes is incomparably more complex, than the simple parity check introducing, for what it is required only introducing in MTD one additional half-adder. It is fair for any ways of implementation of cascade coding with MTD.

The introduced data on usage nonbinary MTD in hardware systems with composite nature of errors also demonstrate its good capabilities.

The offered methods construction of new code classes allow to solve problem of further coding efficiency increase with both concatenated, and nonconcatenated MTD circuits. Though these methods in themselves give a small increment of CG as achieved to customary MTD, amounting usually 0,3 ÷ 0,7 dB, their application allows even more to come nearer to a limit $\mathbf{R} \approx \mathbf{C}$. Let's mark again, that absolute majority of all other coding methods in this very high noise area are become disabled.

The presented research data were successfully utilized, particularly, for creation a special LSI circuit for implementation the MTD decoder.

CHAPTER 7
PROSPECTS FOR THE DECODING ALGORITHMS DEVELOPMENT

A huge codes community and methods of their decoding, and also even more vast set of communication channels types and fast grooth of technological capabilities strongly increase possible complexity of algorithms. It is especially difficult, that the always probable severe change of understanding in a problem of realization, as it already was, for example, after discovering Viterbi algorithm and concatenated codes.

The set up above causes have resulted in attempt to evaluate prospects of those or these directions of coding equipment design, outgoing from idea of a high code gain maintenance at minimum delay of decision making. As the main effectiveness criterion of coding by comparing of supplied CG, and its decoding algorithm further is still considered, as it is usually done, at mean probability of an error per bit $\mathbf{P_b}$ **(e) =** $\mathbf{10^{-5}}$, i. e. under enough representative standard conditions.

Doubtlessly, one more yardstick of selection of algorithm appears its throughout, determining speed, with which one the decoding algorithm can process a flow of coded datum. At creation of throughout for soft versions of decoders it is necessary to return preference, probably, to methods with maximum productivity, as it is possible to consider, that the RAM in modern digital engineering usually is not the limiting factor any more, as well as the sizes of the used programs for error correction.

Certainly, though the delay of decoding actually is relevant at the solution of equipment design problems. It is necessary to get equipment uniform, that is friendly in diagnostics and repair, and also enables analysis by staff of principles of decoders operation.

Selected approach to matching on the basis of delay allows to look at methods of decoding, abstracting from a problem of technological development. It will allow and hereafter to conduct such matching of different methods of decoding, the urgency which one will not depend on a technological level. It is especially convenient, that at any approaches to a problem of realization the complexity of MTD construction always remains comparable with complexity of customary TD. It will allow in all cases, when delay of the solution for MTD and any of diverse method will be comparable, the doubtless simplicity of MTD implementation always will be severe argument for the benefit of its selection.

It is possible to make the main conclusion, that can be done on the basis of matching the main known algorithms/ It is that the MTD has delay of decoding comparable to other high-performance algorithms, or, in some cases, a little bit greater, than it is necessary at usage of other methods with admissible realization complexity.

CONCLUSION

In the book submitted to readers t the analysis and the detail presentation of principles and methods of multithreshold decoding of linear codes for binary, nonbinary and erasure, systematic both nonsystematic block and convolutional codes are offered. Set of the theoretically reviewed properties, outcomes of estimations and the simulations of working MTD allow to consider that the new effective approach to a problem of high noise immunity maintenance is offered for digital data transmission in very noisy channels.

New analysis of the causes of an error propagation effect in decoders is fulfilled. The programs for codes construction with a small level of an error propagation effect are designed, that have allowed to create codes ensuring high characteristics of MTD decoding.

They have studied in detail MTD for block and convolutional, binary and nonbinary systematic codes in different channels with binary, nonbinary and multipositional modulation, and also in channels with erasures. The considerable efficiency of a method of multithreshold decoding ensuring a code gain characteristics, comparable or higher, than in case of other most effective procedures usage: Viterbi algorithm, decoders for concatenated codes and decoders for RS codes.

MTD is at present most simple of known effective decoding procedures.

The fundamental states about high performance MTD are simple enough and clear, that is essential at training to new coding methods for the students and specialists.

The fulfilled projects with simple and effective type MTD decoders, and also successful this algorithm translation at technology template C-MOS LSI circuits are really conditioned by considerable technical and technological advantages MTD with respect to other methods, simple structure and very small realization complexity of this decoder both for block, and for convolutional codes at simultaneous maintenance of a high level of code gain.

The represented data about successful usage MTD for simultaneous fulfillment of functions of a channel and source coding, also considerably extend its application.

The basic outcomes of submitted research for channels with additive white Gaussian noise already now can be utilized in any composite data-transmission systems on the basis of the available datum on MTD capabilities and knowledge of code properties or after some small in its volume jobs to realize an adaptation algorithm to the requirements of concrete developments.

It is represented rather verisimilar, as for many channels with composite nature of errors MTD also will be effective enough, for example, at one of decoding steps in concatenated circuits or in systems with interleaving. The MTD characteristics, reached to the present time, allow to confirm , that at difference of a demanded power scoring of coding gain, for example, for satellite channels from a theoretically bounding level by 2 ÷ 2,5 dB and at quality of decoding very close to optimum, the complexity of MTD appears at the order of magnitude actually at the same level, as complexity of the customary threshold decoder. The further reduction of a difference in trasmitting power approximately by 0,5 ÷ 0,7 dB, results in necessity of sharp increase of length of a code and number of iterations of decoding with some complicating of a function of threshold decoder scheme.

Further improvement of the characteristics MTD even by some tenth parts of a decibel demand even more considerable computing costs at simultaneous additional growth of code length. But the same problems arise and for all other procedures, which one can claim for high performance at code rates, which are enough close to a channel capacity **C**.

It seems doubtless, that problems of decoding implementation for area **R** ≈ **C**, when ones of percents of a transmitted power or frequency band are economized only, probably, and will be a subject of the technical requirements analysis and technological capabilities in communication systems during the nearest years.

The solution of wide application MTD problem in communication systems will be even more advanced forward after realization of researches on usage of this algorithm together with special methods of modulation and other achievements of the information theory and transmission engineering.

It will allow alongside with the already reached result, to supply high performance economic communication for wide and special purpose systems under condition of a huge volume of the transmitting data when there are a rising deficit of channels power and frequency band.