The Multithreshold Decoder Performance in Gaussian Channels

(Report at 7-th ISCTA'03 Ambleside, UK)

(V.V.Zolotarev, Space Research Institute, Moscow, Russia,
e-mail: zolotasd@yandex.ru)

Abstract. The principles of operation and performance of multithreshold decoders (MTD) are reviewed in the field of high levels of a channel noise. These methods in many cases are so effective, as optimum decoding procedures with total search. The complexity of their implementation is practically linear on code length. The comparison of MTD on number of operations with turbo codes is given too.

1. Introduction

The advance in technology of decoding of noise proof codes within many decades surprisingly was not connected in any way to methods of the solution of a functional optimization problem for many discrete variables. Nevertheless decoding, i.e. search of the unique code word among exponentially large number of the possible messages, would be pure naturally to esteem from such stands. However, the majority of decoding algorithms developed before has not used any way for a search of the best decoder solutions of the well-known miscellaneous potent optimization procedures, which one could be applied to search the code words located on minimally possible distance to the received word. Let's remark, that widely applicable in a communication engineering Viterbi algorithm (VA) used for short convolutional codes decoding on a maximum of likelihood, nor falls into to the class of optimization procedures, as it directly searches for the optimum solution on the basis of convenient in implementation total search algorithm.

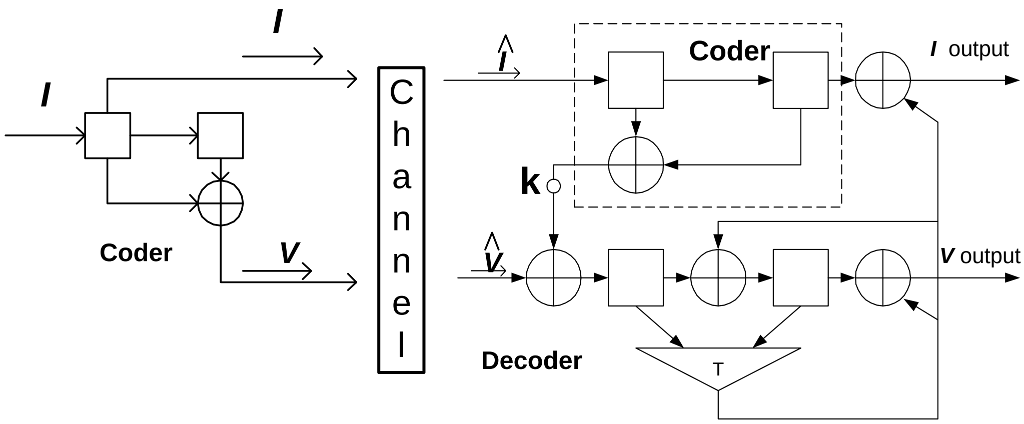
At the same time some algorithms of decoding, in particular, threshold decoders (TD) [1], realizing the elementary error correcting methods, have similar properties, which one are indispensable for implementation valuable effective and simultaneously extremely simple optimization decoding procedures.

2. Optimization idea

Let us consider an example of the simplest encoder/threshold decoding system with code rate R=1/2 and minimum code distance d=3, as it is shown below.

As it follows from an appearance of the encoder and elementary decoder correcting single error, the precise copy of the encoder is created in the decoder too, which one form estimations of code check bits with received from a channel (probably) erroneous information symbols **^*I*** of a code. These symbols appear in the decoder's point **K** and then, after addition at the half-adder with check symbols, received from a channel, **ˆ*V*** will form characters of a syndrome vector **S**, which one depends only on a channel error vector. These characters also move then at threshold switch of the decoder from the syndrome register, as shown at figure.

Even the shape of TD allows to find a simple way of organization of correct optimization procedure, i.e. search the best possible decoder solution. Let's indicate for this purpose the fact, that has never been marked before: in the syndrome register of the decoder there is a difference on check symbols between received with channel distortions vector Ă and such code word Ār, with information symbols which one coincide with an information part of vector Ă received from a channel.



It means, that the total difference between the code word - current hypothesis-solution of the decoder Āi about the transferred code word and received noisy vector Ă will be in such decoder, where in TD single vector will be added else, which one always should correspond to a difference between Ă and Āi - current decoder hypothesis on information characters, which one can change. Such decoder also will contain a current value of a full difference and, therefore, full distance between the solution of the decoder and received vector.

3. New decoding

This exact approach to a problem of high-performance decoding is also the basis for development special multithreshold decoders (MTD) [2-5] since 1974, almost coinciding with TD and the same simple in implementation, as well as their prototype. The MTD's decisions at each decoding step strictly come closer to the solution of the optimum decoder (OD), providing continuation of this process even after several tens attempts of the code block or convolution code symbol correction. Certainly, for the maintenance of high MTD's performances at large channel noise level it is always necessary to select for this procedure only codes, specially constructed for it, with a minimally possible extent of an error propagation effect (EP) [2,3].

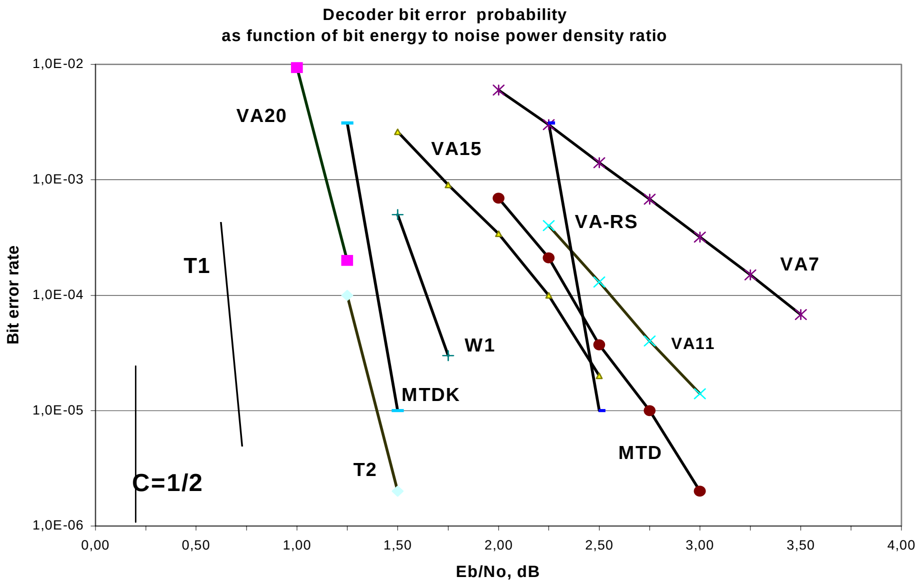
4. Performance

The performance MTD both at independent usage, and in concatenated code designs are comparable to VA's and different concatenated circuits capabilities. In a following figure dependencies are submitted in a conventional form of efficiency of the basic algorithms of decoding for rate **R**=1/2 closed to an decoder error probability per bit **Pb(e)**~10^{-5} as a function from desired values of mean energy attitudes per bit to a power noise density **Eb/No** ratio.

The capabilities of decoders realizing Viterbi algorithm (VA) are shown for different lengths of the encoding register **K**=7, 11, 15 and 20 conforming signs VA***K***. As it follows from their shapes, exponential growing with increase of code length K complexity of decoding for these decoders does not entail to proper growth of energetic efficiency. For example, VA with **K**=20 is more, than in $2*10^4$ times more complex, than standard VA with **K**=7. But its performance still are far from marked at figure vertical line at **Eb/No**=0,2 dB, corresponding capacity of binary Gaussian channel with coherent PSK, equal **C**=1/2.

The result for concatenated circuit with VA and Read-Solomon code curve VA-RS gives. The best turbo code with R=1/2 and the length code n=130'000 bits is presented with line T1 [6], cascaded "woven" code with n=2000 bits - curve W1 [7], and turbo code of length 3570 [8] - curve T2.

The capabilities of simple MTD in Gaussian channel are submitted by the line MTD for a convolutional code with total delay L~8'000 code bits, and concatenated MTD with codes with check parity control (instead of RS codes for other concatenated circuits - only single mod2 adder must be included in coder for this MTD!!!): MTDK with L~10'000 code bits. The properties of MTD admit some additional delay decreasing comparable to mentioned above.



5. Complexity

The main MTD's advantage is the lowest complexity of decoding consisting, as well as in case of customary TD, in summation of weighted checks, matching with a threshold and further decoding symbols and checks change, if this threshold was exceeded. The number of iterations of decoding **I** in this case is no more than 30, and general MTD decoding complexity is estimated for **d**<25 as **N**1~(**d**+2)* (**I**+4). If under the same conditions the performance degradation MTD approximately on 0,1 d**B** channel energy is possible, that is usually could be admitted, the calculus in **MTD** are else more simplified: **N2**~4***d**+3***I**.

Let's point out, that formally defined complexity as number of operations for MTD is approximately on 2 decimal order (~100 times!) less, than for turbo like codes with comparable energetic efficiency. It is essential, that at such estimations the especial difficulty of a certain part of operations, for example, taking the logarithm executed at decoding turbo codes, has not been discussed. Naturally, that by efforts of large number of the specialists engaging these codes, many turbo similar algorithms for 10 years of their intensive research were advanced and simplified. But thus the energetic efficiency of new versions of decoders for these codes is usually a little reduced also. The current complexity of decoders of this class for good energy parameters makes about ten thousands (!!!) operations per every decoded bit.

6. Realization

At proper designing MTD the function of summation of checks easily can be hardware realized in such a manner that it will be completely equivalent to simplest momentary adder. In this case it will appear, that the advantage MTD on operations number concerning other methods already exceeds 3 decimal orders. As MTD enables so easy multisequencing of operations, as well as VA, even the application of the most simple means of parallel calculus allows easily to create and such MTD versions, which ones on each clock tick of the device activity decode till some bits of an information flow both in block, and in convolutional versions of the implementation. It is doubtless, that any other algorithms of decoding never will approximate to this level of MTD complexity.

The comparison MTD and VA also has occurred, that at delay of decision making, in 3-5 times larger, than it is required for an VA, MTD provides the same performance on channel energy for all lengths of the encoding register for VA not superior values K=14. Moreover, in many cases, for example, at admissibility of concatenation even with the simplest parity check control codes, the performance of MTD can be still essentially improved. Let's mark, that the number of operations in MTD remains and in this case very small, whereas VA - total searching algorithm.

7. Conclusion

Passing to conclusions we shall point out that all very high parameters of decoding with MTD are provided by application extremely simple, but resulting and a very potent procedure of functional optimization on the basis of majority decoding algorithms. Note that the real optimum decoding was possible earlier only to short codes and Viterbi algorithm. Thus there is relevant not only simple in the essence a modification of majority decoding, but also difficult mandatory selection of rather specific codes of the self-orthogonal class, which one provide the high characteristics at usage MTD due to their very small susceptibility to an error propagation effect when decoding. Besides there is not less essential moment for successful development of MTD methods, that at creation of these decoders a obligatory and very effective design stage is always the optimization of all parameters of such decoder: threshold values, checks weights, calculation rules for auxiliary functions and reliabilities of checks. The number of optimized MTD parameters sometimes can reach several hundreds. Therefore at creation MTD last stage can demand sometimes up to 99% of CPU time of the special automated designing complex of these high-performance decoders creation. It is extremely important, that after completion of all making stages of a MTD's designing phase this decoder still remains to be the simplest receiving data flow processing scheme with a very fast response time. But choosing of unique values of its parameters, most precisely conforming to a concrete decoder, allows to receive rather considerable additional code gain without redundant hardware and any additional computing costs. Certainly, last two circumstances could not be taken into account so many years ago, when some completely fair results about low efficiency of simple repeated decoding attempts on the basis of the majority schemes were published (see, for example, [9] and the other ones).

Huge advantage MTD before all other decoding circuits on operation number, the estimations of which were given above, many years ago published outcomes on non-binary character data decoding [10], multipositional modulation systems [11], and also other capabilities of this very potent method allow to state, that all principled problems of attempted research are successfully resolved at the present moment. Thus, after 30-year's researches the wide range of multithreshold algorithms is designed, which one can be recognized by the main coding method for many modern high speed communication systems with extremely high levels of code gain and very large throughput.

The report is accompanied by demonstration computer movies about multithreshold decoding, illustrating all steps of MTD work in convenient and pictorial form.

8. Acknowledgements

This investigations were supported by Russian Academy of Science, Council on complex problems of cybernetics RAS, Space Research Institute RAS (SRI RAS), Ministry of Telecommunication and informatization RF and Radio research&development institute (NIIRadio) of this Ministry.

For additional information about MTD: in Moscow, Russia: in SRI RAS: tel. work - +7 095 333 23 56, fax +7 095 333 21 77, in NIIRadio: +7 095 261 03 27, e-mail: niir.189@mail.ru , tel. home +7 571 51 32, mob.: +7 916 518 86 28,
e-mail: zolotasd@yandex.ru

Literature

1. Massey J. Threshold decoding. - M.I.T. Press, Cambridge, Massachusets,1963.

2. Samoilenko S.I., Davidov A.A., Zolotarev V.V., Tretjakova E.I. Computer Networks. - Moscow, Science ("Наука"), 1981, p. 277.

3. Zolotarev V.V. Effective Multithreshold Algorithms of Decoding. - AS USSR, Scientific Council on Integrated Problem "Cybernetics", preprint, Moscow., 1981, p. 75.

4. Zolotarev V.V. Usage of Noise proof Coding in a Communication Engineering. - "Electrosvjaz", Moscow, №7, pp.7-10, 1990.

5. Zolotarev V.V. Real Code Gain for Satellite Channels. - In Proceedings: 4-th International Conference " Satellite Communications - ICSC-2000 ", Vol.2, pp. 20-25, Moscow, MCNTI, 2000, pp. II-20 - II-25 .

6. Berrou C., Glavieux A., Thitimajshima P. Near Shannon Limit Error-Correcting Coding and Decoding: Turbo-Codes. - Proceeding of ICC ' 93, Geneva, Switzerland, pp. 1064-1070, May 1993.

7. Host S., Johannesson R., Zyablov V. A first encounter with binary woven convolutional codes. - in Proc. International Symposium on Communication Theory and Applications, pp. 13-18, Lake District, UK, July 1997.

8. Andrews K., Berner J., Chen V., Dolinar S., Pollara F., Stanton V. Turbo-decoder implementation for the deep space network. - IPN Progress Report 42-148, Feb. 15, 2002.

9. Townsend R.L., Weldon E.J. Self-Orthogonal Quasi-cyclic Codes. IEEE Trans., IT-13, 1967, pp.183-195.

10. Zolotarev V.V. Coding Algorithms for the Character Data in Computer Networks. - In book: " Problems of Cybernetics ", VK - 106, Moscow, 1985.

11. Banket V.L., Zolotarev V.V. Efficiency of Multipositional Modulation Systems and Multithreshold Decoding. - In:" 9 All-Union school-seminar on computer networks ", part 3.2, Moscow - Puschino, 1984.